Technische Universität Berlin Winter 2025/26
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Graph Theory (DS II) - Sheet 11

Exercise 11.1.

We call two drawings of a connected planar graph *d-isomorphic* if their dual graphs are isomorphic. A rotation system describes the combinatorics of a drawing by the cyclic permutation of incident edges of each vertex. We call two drawings of a planar graph r-isomorphic if (there is a permutation of the vertices and edges such that) the rotation systems agree or are the reverse of each other. Show that these two notions are different and that one implies the other.

Exercise 11.2.

Let X be a k element point set on the unit circle and $\bar{X}$ the set of antipodal points on the circle such that $X \cap \bar{X} = \emptyset$. How many triangles built from points in $X \cup \bar{X}$ contain the midpoint of the circle? (If the midpoint lies on one of the edges of the triangle, it is considered outside of the triangle)

Exercise 11.3.

Prove that the following is equivalent:

(a) G is outerplanar.

(b) G has no subdivision of K_4 or $K_{2,3}$ as a subgraph.

(c) G does not have K_4 or $K_{2,3}$ as a minor.

Exercise 11.4.

Read the proof of the following statement in the script or some other source: Any 3-connected graph G on at least 5 vertices has an edge e such that G/e is still 3-connected (G/e is the graph resulting from G by contracting e).

Exercise 11.5.

Which of these sets of graphs is minor closed? For those sets that are minor closed, find a minimum set of forbidden minors which characterises the set.

(a) k-connected graphs for some $k \in \mathbb{N}$.

(b) Forests.

(c) Linear forests. Forests are linear if their connected components are paths.

(d) Graphs with degeneracy at most d for some $d \in \mathbb{N}$. A graph G has degeneracy d if any subgraph of G contains a vertex of degree at most d.