Technische Universität Berlin Winter 2025/26
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Graph Theory (DS II) - Sheet 9

Exercise 9.1.

(a) Find a planar simple graph G that is 3-edge-connected and whose dual graph is not unique.

(b) Find two different plane graphs with the same dual.

(c) Let G be a plane graph. Show that if the connectivity of its dual is at least 2, then G has at most one component that is not a tree.

Exercise 9.2.

A planar simple graph G is a triangulation, if a drawing without crossings of G exists, such that every face has degree 3 (even the outer face). Show that

(a) Every planar graph on at least 3 vertices is a spanning subgraph of a triangulation.

(b) Every simple planar graph is an induced subgraph of a triangulation.

(c) Triangulations (on more than 3 vertices) are 3-connected.

Exercise 9.3.

(a) Show that for all graphs G on at least 11 vertices, either G or its complement is not planar.

(b) Find a planar graph without a vertex of degree 4 or less.

(c) Find a bipartite planar graph without vertices of degree 2 or less.

Exercise 9.4.

A pseudoline is a simple curve $\gamma\colon \mathbb{R} \to \mathbb{R}^2$ with $\lim_{x\to\pm\infty} |\gamma(x)| = \infty$[a]. An arrangement of pseudolines is a finite set of pseudolines, such that any pair crosses in exactly one point and they have no further intersections. We also require that no three pseudolines cross in the same point, see Figure 1. Given an arrangement of pseudolines, they cut the plane into regions. If there are n pseudolines, how many regions is the plan cut into?

[a] Without any weird topology like infinite spirals etc., think combinatorial.

Bonus Exercise

For which n is the following possible. Choose pairwise different numbers $a_1, \ldots, a_n \in [n+1]$ such that the differences

$$|a_i - a_{(i+1 \mod n)}|$$

are also pairwise different.

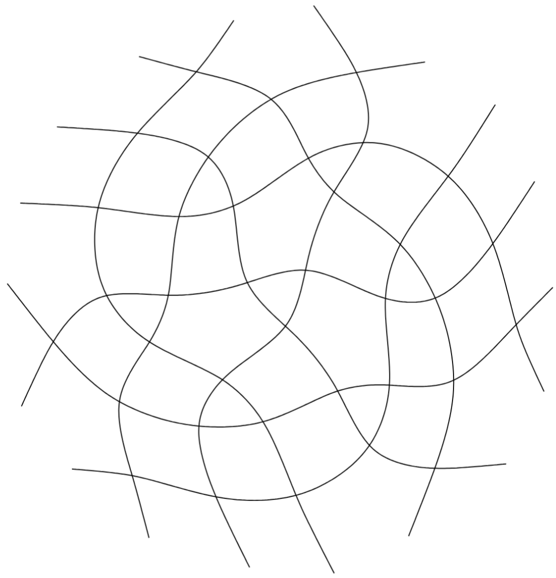

Figure 1: An arrangement of 9 pseudolines. Imagine that the ends of the lines extend to infinity in both directions without further crossings.