Technische Universität Berlin
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Graph Theory (DS II) - Sheet 8

Exercise 8.1.

Prove that $R(k, \ell) \leq R(k-1, \ell) + R(k, \ell-1)$. Use this to give a better upper bound for $R(k, k)$ than the one from the lecture.

Exercise 8.2.

Let $R(G, H)$ be the smallest integer R, such that any red-blue coloring of the edges of K_R contains a red subgraph G or a blue subgraph H.

(a) What is $R(K_{1,m}, K_{n,1})$?

(b) Show that $R(C_4, C_4) = 6$.

Exercise 8.3.

Do all of the domino tilings of the following boards have break-lines?

(a) $4 \times k$.

(b) $6 \times k$.

(c) Characterize all values k, ℓ such that the $\ell \times k$ board has a break line for all domino tilings.

Exercise 8.4.

Prove that for any $n \in \mathbb{N}$, there is an $N \in \mathbb{N}$ big enough, such that no matter how you partition $[N]$ into n parts $P_1, \ldots, P_n$, there will be a triple $x, y, z \in P_i$ in one of the parts, such that $z = x + y$.

Exercise 8.5.

(a) Show that $R(3, 4) \leq 10$.

(b) Show that $R(3, 4) \leq 9$.

(c) Show that $R(3, 4) = 9$,

Bonus Exercise

Color the point of $\mathbb{N}^2$ in the plane with r different colors. Is there always an isosceles triangle with one horizontal edge and one vertical edge whose vertices all have the same color?