Technische Universität Berlin Winter 2025/26
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Graph Theory (DS II) - Sheet 4

Exercise 4.1.

Given a graph $G = (V, E)$ and two spanning trees $T_1 = (V, E_1)$ and $T_2 = (V, E_2)$ of G. Show that for every edge $e_1 \in E_1 \setminus E_2$ there is an edge $e_2 \in E_2 \setminus E_1$ such that $T_1' = (V, (E_1 \setminus \{e_1\}) \cup \{e_2\})$ and $T_2' = (V, (E_2 \setminus \{e_2\}) \cup \{e_1\})$ are also both spanning trees of G.

Exercise 4.2.

Let X be a set of n points on a circle. Let D_n be a drawing of K_n on X with straight line edges. A non-crossing spanning path P_n of D_n is a spanning path in the corresponding K_n such that no edges of P_n cross in D_n. Show that if n is even, the edges of K_n can be partitioned into non-crossing spanning paths.

Exercise 4.3.

Show that for any tree T with t edges,

$$\frac{t-1}{2}n - o(n) \leq \text{ex(n, T)}.$$

If $T = K_{1,t}$, show that the bound is tight up to $o(n)$.

Exercise 4.4.

Let G be a n-vertex graph with degree sequence $(d_1, \ldots, d_n)$. Assume that

$$\sum_i \binom{d_i}{2} > (m-1)\binom{n}{2}.$$

Show that G contains $K_{2,m}$ as a subgraph.

Bonus Exercise

You invite your n friends to a dinner party. You have planned a seating arrangement around your circular table and have placed little name cards so that everybody knows where to sit. Unfortunately, your friends completely ignore the cards and just sit down (rude). You don't want to seem like a control freak, so you don't say anything. But before the dinner is served, you manage to get the chance to spin the table without anybody seeing. You spin it into the position such that the maximum possible number of your friends sit in front of the correct card. In the worst case (and depending on n), how many of your friends will sit in front of the correct card after your spin?