

Graph Theory (DS II) - Sheet 0

Exercise 0.1.

Anna and Bob sit in front of a row of n coins of possibly differing values. They play a game where Anna and Bob choose one coin from one of the ends of the row after each other (Anna starts). Show that

- (a) Anna can win at least half the total value of the coins if $n = 100$.
- (b) There is a set of coins and an ordering such that Anna can not win a $\frac{1}{100}$ -fraction of the total value if $n = 101$.

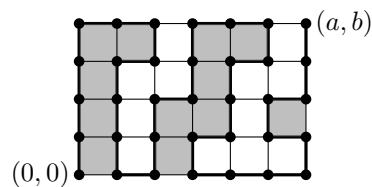


Figure 1: A gridpath from $(0,0)$ to (a,b) touching each vertex exactly once. The squares reachable from the bottom or right are shaded.

Exercise 0.2.

Consider a $a \times b$ grid and a grid path from $(0,0)$ to (a,b) which touches each vertex exactly once, see Figure 1. Shade the squares which can be reached from the bottom or the right without crossing the path. Show that exactly half the squares are shaded.

Exercise 0.3.

You have thirty coins, from which 10 are heads up and 20 are tail up. You want to make two sets of coins without feeling the coins nor looking at the coins, such that each pile contains the same number of coins which are heads up. What do you do?

Exercise 0.4.

Two thieves have stolen a necklace with $2m$ golden and $2n$ silver beads. Show that the necklace can be cut into two pieces such that each piece consists of m golden and n silver beads. (Independent of the ordering of the beads!) Why are there two antipodal points on earth with the same temperature and the same pressure? What is the connection between these two questions?