9. Practice sheet for the lecture: **Felsner/ Wesolek**
Graph Theory (DS II) 12. December 2023

Due dates: 19./21. December
https://page.math.tu-berlin.de/~felsner/Lehre/dsII23.html

(1) Find a 2-connected graph which is not Eulerian, whose line graph[1] is Hamiltonian.

(2) Suppose G is an n-vertex graph with $\delta(G) \geq \frac{n}{2} + 1$ and let $v_1, v_2, v_3 \in V(G)$ be pairwise distinct. Suppose G' is the graph obtained from G by adding two vertices x_1, x_2 and connecting x_1 to v_1, v_2 and x_2 to v_2, v_3. Show that G' is Hamiltonian.

(3) Let T be a tree on at least 3 vertices. Show that for any edge $xy \in E(T)$ there is a Hamilton cycle in T^3 which contains the edge xy.[2]

(4) A permutation is a Baxter permutation if there are no indices $i < j < k$ with $\pi_{j+1} < \pi_i < \pi_k < \pi_j$ nor with $\pi_j < \pi_k < \pi_i < \pi_{j+1}$.

   (a) Show that the family of Baxter permutations of length n is a zigzag family.

   (b) Research which objects are counted by Baxter numbers.

(5) Let the binary code $v_A = (a_1, \ldots, a_n)$ of a set $A \subset [n]$ satisfy $a_i = 1$ if and only if $i \in A$. Let $S = \binom{[n]}{k}$. Show that there is a sequence of the vectors $\{v_A \mid A \in S\}$ such that consecutive vectors differ in exactly two entries.

[1] The line graph H of a graph G is the graph with $V(H) = E(G)$ and two vertices are connected in H if the corresponding edges in G are incident.

[2] T^3 is the graph with vertex set $V(T^3) = V(T)$ and edge set $E(T^3) = \{\{u, w\} \mid 1 \leq dist_T(u, w) \leq 3\}$.