7. Practice sheet for the lecture: **Felsner/ Wesolek**
Graph Theory (DS II) 28. November 2023
Due dates: 05./07. Dezember
`https://page.math.tu-berlin.de/~felsner/Lehre/dsII23.html`

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(1)
  (a) Show that $R(3,4) \leq 10$.
  (b) Improve (a) to $R(3,4) \leq 9$.
  (c) Show that $R(3,4) = 9$.

(2) Let $R(G,H)$ be the smallest integer R such that any red-blue coloring of the edges of K_R contain a red subgraph G or a blue subgraph H.
  (a) What is $R(K_{1,m}, K_{1,n})$? (exactly, for any given $m, n \in \mathbb{N}$)
  (b) Show $R(C_4, C_4) = 6$.

(3) Prove that for any $n \in \mathbb{N}$ there is an $N \in \mathbb{N}$ big enough such that no matter how you partition $[N]$ into n parts $P_1, \ldots, P_n$, there will be a triple $x, y, z \in P_i$ for some i such that $x = z + y$. [Hint: Colour edges, $N = R_2(n; 3, 3, \ldots, 3)$ is big enough.]

(4) Show that if $N \geq R_3(2; t, t)$, then any set of N points in general position contains a subset of t points in convex position. [Hint: Consider triplets of points and the order in which they appear in clockwise direction.]