6. Practice sheet for the lecture: **Felsner/ Wesolek**
Graph Theory (DS II) 20. November 2023

Due dates: 28./30. November
`https://page.math.tu-berlin.de/~felsner/Lehre/dsII23.html`

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(1) Recall that the edit distance d_E between two graphs G, G' on n vertices is defined as

$$d_E(G, G') = \frac{1}{n^2} \sum_{i,j=1}^{n} |A(G)_{ij} - A(G')_{ij}|.$$

Prove that for the cut distance $d_\square$ it holds that

$$d_\square(G, G') \leq d_E(G, G').$$

(2) Find the limit of the graph sequence $G_1, G_2, \ldots$

(a) G_n is a tree on n vertices (here we have some fixed tree for every $n \in \mathbb{N}$).

(b) Let $0 < r < 1$ be fixed. Let G_n be the n-vertex graph with $i \sim j$ if and only if $|i - j| \leq rn$.

(3) This exercise is to show that for a tree T on t edges it holds that $ex(n, T) \leq (t-1)n$.

(a) Show that a graph with average degree d contains a subgraph with minimum degree greater than $\frac{d}{2}$.

(b) Show $ex(n, T) \leq (t-1)n$.

(4) Use Zykov-symmetrization to show the following. Let $n \geq k+1$. Then

$$ex(n, K_3, K_{k+1}) = \#\text{ triangles in } K_{n_1,\ldots,n_k},$$

for some integers $n_1, n_2, \ldots, n_k \geq 0$ with $n = \sum_{i=1}^{k} n_i$.

[Hint: `http://www.sfu.ca/~agwesole/Hinweis3`]

(5) Let $G = (V, E)$ be a graph. The Wei inequality says $\alpha(G) \geq \sum_{v \in V(G)} \frac{1}{d_v+1}$. Characterize the graphs with equality $\alpha(G) = \sum_{v \in V(G)} \frac{1}{d_v+1}$.

(6) [Bonus[1]] Show that for every $\varepsilon > 0$ there is some $\varepsilon' > 0$ such that if G is an N-vertex graph with fewer than $\varepsilon' N^4$ copies of K_4, then G can be made K_4-free by removing at most εN^2 edges.

[1] This question is completely optional. It should help you understand the lecture better, so I recommend going through it.