(1) Suppose P is a path between two vertices u, v in a graph G with $\kappa(G) \geq 2$. Prove or disprove: There exists a path Q which is internally vertex-disjoint from P.

(2) Suppose G has $\kappa(G) = 2$.

   (a) Let G' be obtained from G by subdividing an edge $e \in E(G)$ (that is, replacing e by a path of length 2, see Figure 1). Show that $\kappa(G') = 2$.

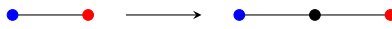   (b) Show that any two edges of G are on a common circle.

Figure 1: Subdividing an edge.

(3) Let T be a tree on $n \geq 3$ vertices and $x_i = |\{v | d(v) = i\}|$.

   (a) Show that

$$\sum_{i=3}^{n-1} (i-2)x_i = x_1 - 2$$

   (b) How many non-isomorphic trees with 5 leaves and no vertex of degree 2 exist?

(4) Let G be a connected graph that does not contain P_4 or K_3 as induced subgraphs (P_4 is a path with 4 vertices). Prove that G is a complete bipartite graph.

(5) The girth of a graph G, denoted $g(G)$, is the length of the shortest cycle in G.

   (a) Show that if an n-vertex graph is d-regular and has girth $g = 2k+1$ for $k \geq 1$, then

$$n \geq d \sum_{i=0}^{k-1} (d-1)^i + 1.$$

   (b) Provide an example to show the bound is tight when $d = 3$ and $g = 5$. [Hint:`http://www.sfu.ca/~agwesole/Hinweis2`]