2. Practice sheet for the lecture: **Felsner/ Wesolek**
Graph Theory (DS II) 24. Oktober 2023

Due dates: 31. October/02. November
`https://page.math.tu-berlin.de/~felsner/Lehre/dsII23.html`

(1)

(a) Prove that the number of students who have an odd number of friends in the "Graphentheorie" Vorlesung is even.

(b) Prove or disprove: If $u \in V$ and $v \in V$ are the only vertices of odd degree in G, then you can walk from u to v in G.

(2) Let $k \geq 2$ be an integer, and let G be a graph with n vertices and n edges with precisely one vertex of degree j for every $j \in [2, k]$ and all other vertices of degree 1. What is n?

(3)

(a) If G contains a **circle**, then G contains a **simple cycle**.
[A circle is a non-empty closed walk (it ends up in the same vertex as it started) that does not use any edge more than once, a simple cycle is a circle, that doesn't use any vertex more than once.]

(b) Let A be the adjacency matrix of G, so $A \in \{0,1\}^n$ with $a_{i,j} = 1$ if and only if $ij \in E(G)$. Show that there is a walk of length k from u to v if and only if $a^k_{u,v} \neq 0$.

(4) Suppose G and H are two graphs on the vertex set V where every vertex in V has the same degree in G and H. Let $G\Delta H$ be the graph on the vertex set V with edge set $E(G\Delta H) = (E(G) \setminus E(H)) \cup (E(H) \setminus E(G))$.

(a) Show that every vertex in $E(G\Delta H)$ has even degree.

(b) Show that $E(G\Delta H)$ can be partitioned into symple cycles. [Hint: `https://www.sfu.ca/~agwesole/Hinweis`]