
**15. Practice sheet for the lecture:
Graph Theory (DS II)**

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Due dates: 13./ 15. February

<https://page.math.tu-berlin.de/~felsner/Lehre/dsII23.html>

- (1) Greedy coloring worst case:
 - (a) Show that there is a bipartite graph on $2n$ vertices and an order of them, such that the Greedy algorithm uses n colors to color it instead of 2.
 - (b) Show that there is a planar graph on 2^n vertices and an order of them such that the Greedy algorithm uses more than n colors to color it.
- (2) A connected greedy coloring of a connected graph G is a greedy coloring with respect to a connected sequence (v_1, \dots, v_n) , that is, the graph on v_1, \dots, v_i is connected for each $1 \leq i \leq n$.
 - (a) Show that a connected greedy coloring always uses 2 colors for connected bipartite graphs with at least one edge.
 - (b) Consider the graph G which contains three graphs G_1, G_2, G_3 as shown below and additional edges between the vertices a_1, a_2, a_3 . Show that any connected greedy coloring of G needs at least 4 colors.

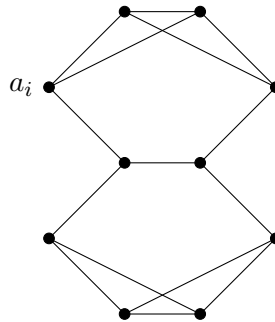


Figure 1: The graph G_i .

- (3) The last part of the tutorial will be to answer easier questions similar to the ones at the beginning of our tutorials and to clarify questions on proofs from the lecture or exercises. For that, please fill out this questionnaire on the topics that you would like to talk about the most:



Figure 2: Or access the link here: <https://votingo.cedis.fu-berlin.de/B7YWLA>