14. Practice sheet for the lecture: **Felsner/ Wesolek**
Graph Theory (DS II) 30. January 2024
Due dates: 06./ 08. February
https://page.math.tu-berlin.de/~felsner/Lehre/dsII23.html

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(1) If $\pi = (\pi_1, \pi_2, \ldots, \pi_n)$ is any permutation of the numbers from $1 \ldots n$, then the permutation graph on π is the graph where $i \sim j$ for $i < j$ if j appears before i in π.

   (a) What does the clique number $\omega(G)$ and the independence number $\alpha(G)$ correspond to in a permutation graph G?

   (b) Give an efficient algorithm to compute $\alpha(G)$.

   (c) Are permutation graphs perfect?

(2) Show that:

   (a) If a collection of (connected) subtrees $T_1, T_2, \ldots, T_\ell$ of a tree T pairwise intersect ($V(T_i) \cap V(T_j) \neq \emptyset$), then they share a common vertex ($V(T_1) \cap V(T_2) \cap \cdots \cap V(T_\ell) \neq \emptyset$).

   (b) Conclude that a graph of treewidth k does not contain K_{k+2} as a subgraph.

(3)

   (a) Show that every graph of treewidth k is a subgraph of a chordal[1] graph G with clique number $\omega(G) = k + 1$.

   (b) Show that every graph with treewidth at most k has a vertex of degree at most k.

(4) Show that there is an algorithm for the maximum independent set problem with running time $2^k \cdot O(n^2)$ when given a graph on n vertices and a tree decomposition of the graph with width at most k and $O(n)$ nodes.
Hint: https://www.sfu.ca/~agwesole/Hinweis7

(5) Suppose T is the 3-ary tree[2] of height h . Show by induction that the pathwidth of T is at least $h - 1$.

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[1] A graph is a chordal graph if it has no induced cycle of length at least 4.

[2] that is the rooted tree such that all vertices at distance at most $h - 1$ from the root have degree 3 and all vertices at distance h are leaves