13. Practice sheet for the lecture: **Felsner/ Wesolek**
Graph Theory (DS II) 23. January 2024
Due dates: 30. January/ 01. February
https://page.math.tu-berlin.de/~felsner/Lehre/dsII23.html

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(1) A graph has page number at most k if there exists an ordering of the vertices $\pi = (v_1, v_2, \ldots, v_n)$ and a partition of the edges $E_1, \ldots, E_s$ such that for all t there is no overlapping pair of edges in E_t (no pair $v_i v_k$ and $v_j v_\ell$ with $i < j < k < \ell$).

  (a) Show that outerplanar graphs have page number 1.

  (b) Show that the page number is not monotone under subdivision, that is, subdividing the edges of a graph might increase its page number.

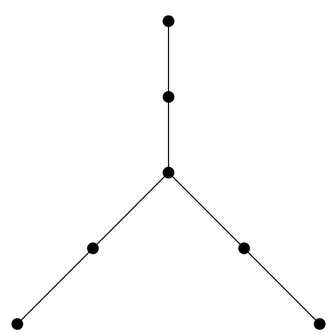
(2) Show that for every set $P \subset \mathbb{R}^2$ of n points in general position (no three points are collinear) any tree T admits a plane straight-line embedding on P.

(3)

  (a) Describe a plane graph G with n vertices that can be embedded (while preserving the outer face) on a grid of size $\frac{2n}{3} - 1 \times \frac{2n}{3} - 1$ but not on a smaller grid.[1]

  (b) Can you draw G on a smaller grid if you are allowed to change the embedding (i.e. outer face)?

(4) Characterize the graphs of dimension 2. A graph is of *dimension at most k* if there exists a realizer of size k, i.e. k orderings of the vertices $\pi_1, \ldots \pi_k$ such that for every edge uv and vertex $w \neq u, v$ there exists an ordering such that u, v both come before w in the ordering. Hint: Show that neither the graph in the below figure nor a cycle have dimension 2.



(5) We say a graph has dimension at most $k - \frac{1}{2}$ if there exists a realizer of size k such that π_2 is the reverse of π_1.

  (a) Characterize the graphs of dimension $1\frac{1}{2}$.

  (b) Show that the graphs of dimension at most $2\frac{1}{2}$ are outerplanar graphs.
  Hint: https://www.sfu.ca/~agwesole/Hinweis6

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[1] Here, a grid of size k contains all the points in $[0, k] \times [0, k]$. Further, we assume that n is divisible by 3.