11. Practice sheet for the lecture: Felsner/ Wesolek
Graph Theory (DS II) 09. January 2024
Due dates: 16./18. January
https://page.math.tu-berlin.de/~felsner/Lehre/dsII23.html

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(1) Show that every planar graph can be decomposed into 5 forests (or less).[1]

(2) Which of these sets of graphs is minor closed? For those sets that are minor closed, find a minimum set of forbidden minors which characterises the set.

  (a) Set of k-connected graphs for some $k \in \mathbb{N}$.

  (b) Set of forests.

  (c) Set of linear forests. Forests are linear if their connected components are paths.

  (d) Set of graphs with degeneracy at most d for some $d \in \mathbb{N}$.

(3) In this exercise you will show Kuratowski- and Wagner-type results for outerplanar graphs. Prove that the following are equivalent:

  (a) G is outerplanar.

  (b) G has no subdivision of K_4 or $K_{2,3}$ as a subgraph.

  (c) G does not have K_4 or $K_{2,3}$ as a minor.

(4) Show that if G is a plane bipartite graph then the dual of G is Eulerian.

(5) Show that if G is an Eulerian triangulation then G is 3-colorable.

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[1] A decomposition of a graph G is a collection of edge-disjoint subgraphs of G such that every edge belongs to exactly one subgraph. In fact, 3 forests are enough by a theorem of Nash-Williams. If you find this question not stimulating enough, show that every planar graph can be decomposed into 3 forests (or less).