
**1. Practice sheet for the lecture:
Graph Theory (DS II)**

Felsner/ Wesolek
13. Oktober 2023

Due dates: 24./26. October

<https://page.math.tu-berlin.de/~felsner/Lehre/dsII23.html>

- (1) Anna and Bob sit in front of a row of n coins of possibly differing values. They play a game where Anna and Bob choose one coin from one of the ends of the row after each other (Anna starts). Show that
 - (a) Anna can win at least half the total value of the coins if $n = 100$.
 - (b) There is a set of coins and an ordering such that Anna can not win a $\frac{1}{100}$ -fraction of the total value if $n = 101$.
- (2) You have thirty 1€ coins¹, from which 10 are heads up and 20 are tail up. You want to make two sets of coins without neither feeling the coins³ nor looking at the coins, such that each pile contains the same number of coins which are heads up. What do you do?
- (3) Two thieves have stolen a necklace with $2m$ golden and $2n$ silver beads. Show that the necklace can be cut into two pieces such that each piece consists of m golden and n silver beads. (Independent of the ordering of the beads!)
Why are there two opposite points on the equator with the same temperature? What is the connection between these two questions?
- (4) From an 8×8 grid of unit squares, remove two squares which are diagonally opposite to each other. Show that the board can not be covered by 2×1 dominoes.
- (5) Ralf and Anna go to a dinner party with $n - 1$ other couples. Each participant shakes hands with every unknown person. Afterwards Ralf does a survey and discovers that everyone of the $2n - 1$ other attendees shook hands with a different number of people. How many people did Anna shake hands with?

¹Which you have been collecting for your next visit to a laundromat.²

²You just moved to Berlin and could not afford to rent an apartment with a washing machine.

³This is possible, since you are wearing gloves.