10. Übungsblatt zur Vorlesung: **Felsner/ Schröder**
Graphentheorie (DS II) 11. Januar 2022

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`http://www.math.tu-berlin.de/~felsner/Lehre/dsII21.html`

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(1) We call two drawings of a connected planar graph *d-isomorphic* if their dual graphs are isomorphic. A *rotation system* describes the combinatorics of a drawing by the cyclic permutation of incident edges of each vertex. We call two drawings of a planar graph *r-isomorphic* if (there is a permutation of the vertices and edges such that) the rotation systems agree. Show that these two notions are different and that one implies the other.

(2) Let X be a k element point set on the unit circle and $\overline{X}$ the set of opposite points on the circle such that $X \cap \overline{X} = \emptyset$. How many triangles built from points in $X \cup \overline{X}$ contain the midpoint of the circle? (If the midpoint lies on one of the edges of the triangle, it is considered outside of the triangle)

(3) Does every planar graph have a circle packing representation, such that all radii are equal? Find necessary conditions for graphs with this properties. Find classes of graphs which have this property.

(4) Crossing number variants:
Let the crossing edge number of a drawing Γ of a graph G be the number of edges in Γ that are crossed, no matter how many times they are crossed. Let $\mathrm{cre}(G)$ denote the minimum of all crossing edge numbers of drawings of G.

(a) Prove that $\mathrm{cre}(G) \leq 2\,\mathrm{cr}(G)$.

(b) Find an example graph G such that $\mathrm{cre}(G) < 2\,\mathrm{cr}(G)$.

(5) Separator theorem lower bound:
Let G be the $[n] \times [n]$ grid, that is, $V = [n] \times [n]$, and there is an edge between (i, j) and (i', j') if and only if $||(i, j) - (i', j')|| = 1$. Let X be a k element subset of $V(G)$, $k \leq \frac{n^2}{2}$. Prove that the number of vertices not in X, but adjacent to some vertex in X is at least $\sqrt{\frac{k}{2}}$.
[Hint: Which rows/columns contain vertices of X?]