7. Übungsblatt zur Vorlesung: **Felsner/ Schröder**
Graphentheorie (DS II) 6. Dezember 2021

Besprechungsdatum: 13. /16. Dezember
`http://www.math.tu-berlin.de/~felsner/Lehre/dsII21.html`

This week's Thursday practice session will take place in MA043 with Prof. Felsner!

(1) Let there be a set of n distinct points in the plane, such that no two are further apart than $\sqrt{2}$. Show that there are at most $(1-\frac{1}{3})\frac{n^2}{2}$ pairs of points, that have distance larger than 1.

(2) A complete multipartite graph $G=(V,E)$ is a maximal graph with respect to the chromatic number, that is for all $e \in \binom{V}{2} \setminus E$ we get $\chi(G+e) > \chi(G)$.
Show that a simple graph is complete multipartite if and only if it contains no induced subgraph with exactly 3 vertices and 1 edge.

(3) Let $G=(V,E)$ be a graph. The Wei inequality says $\alpha(G) \geq \sum_v \frac{1}{d_v+1}$. Characterize the graphs with equality $\alpha(G) = \sum_v \frac{1}{d_v+1}$.

(4) Let $R(G,H)$ be the smallest integer R, such that any red-blue coloring of the edges of K_R contains a red subgraph G or a blue subgraph H.

  (a) What is $R(K_{1,m}, K_{1,n})$? (exactly, for any given $m,n \in \mathbb{N}$)

  (b) Show $R(C_4, C_4) = 6$.

(5) Do all of the domino tilings of the following boards have break-lines?

  (a) $4 \times k$-board

  (b) $6 \times k$-board

  (c) Characterize all the values k and l, such that the $k \times l$-board has a break-line.