2. Übungsblatt zur Vorlesung: **Felsner/ Schröder**
Graphentheorie (DS II) 2. November 2021
Besprechungsdatum: 8. /11. November
`http://www.math.tu-berlin.de/~felsner/Lehre/dsII21.html`

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(1) Degree sequences

(a) Show that $(d_1, ..., d_n)$ with $d_i > 0$ for all $i = 1, \ldots, n$ is the degree sequence of a tree, if and only if $\sum_{i=1}^{n} d_i = 2n - 2$.

(b) Let $d_1 < d_2 < \ldots < d_k$ be natural numbers. Show that a graph G with $d_k + 1$ vertices exists, such that $\{d_1, d_2, \ldots, d_k\}$ is the set of degrees of G.

(2) Connectivity

(a) Let H_d be the d–dimensional Hypercube. Show $\kappa(H_d) = d$.

(b) Show, that in a k–connected graph with n vertices, there are at least $\lceil \frac{kn}{2} \rceil$ edges. Actually, there are graphs with exactly this many edges for almost all pairs (k, n) with $k \leq n$. Show this for as many pairs as possible.

(3) Radius, diameter, girth

(a) Prove that $\mathrm{girth}(G) \leq 2\,\mathrm{diam}(G) + 1$ for every graph which has at least one simple cycle.

(b) Do some research on the topic of Moore graphs, in particular, give some examples, show the relation with (a) and prove at least one other interesting property that they have.

(c) Show that the inequality $\mathrm{rad}(G) \leq \mathrm{diam}(G) \leq 2\,\mathrm{rad}(G)$ is tight by giving for $k \in \mathbb{N}$ and both inequalities one graph G each, such that $\mathrm{rad}(G) = k$ and such that G fulfills the inequality as an equality.

(4) Let T be a tree with $n \geq 3$ and $x_i = |\{v \mid d(v) = i\}|$.

(a) Show $\sum_{i=3}^{n-1} (i - 2)x_i = x_1 - 2$

(b) How many different (non-isomorphic) trees with 5 leaves and without vertices of degree 2 are there?

(5) Prove the undirected p-q edge version of the Theorem of Menger.
[Hint: Use the undirected p-q vertex version from the lecture]