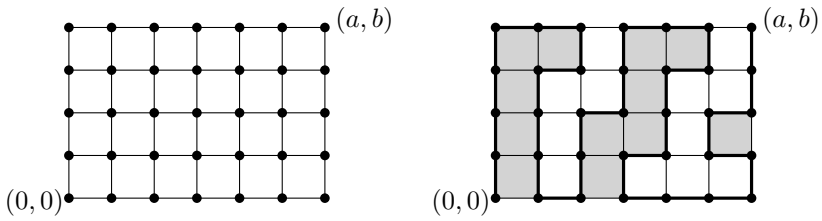
1. Practice sheet for the lecture: **Felsner/ Kleist**
Graph Theory (DS II) 12. October 2015

Due dates: 19.-20. October
`http://www.math.tu-berlin.de/~felsner/Lehre/dsI15.html`

(1) Two thieves have stolen a chain with $2m$ gold and $2n$ silver beats. Show that the chain can be cut into two pieces such that each piece consists of m gold and n silver beats. (Independent of the ordering of the beats!)
Why are there two points on the equator with the same temperature? What is the connection between these two questions?

(2) Consider a planar graph with vertices on integer coordinates $0 \leq x \leq a$, $0 \leq y \leq b$ and edges between pairs of vertices with distance 1, see the figure below. Let P be a path from $(0,0)$ to (a,b) visiting all vertices exactly once. P partitions the set of boxes into two sets: the set reachable from top or left and the set reachable from bottom or right. Show that the sets have the same cardinality.
In the right figure the set of boxes reachable from bottom and right is shaded.



(3) (*written*) Ralf and Anna go to a dinner party with $n-1$ other couples. Each participant shakes hands with every unknown person. Afterwards Ralf does a survey and discovers that everyone of the $2n-1$ other attendees shook hands with a different number of people. How many people did Anna shake hands with?

(4) How many paths from x_0 to v_n exist? [Hint: Tools from Combinatorics are helpful.]