15. Practice sheet for the lecture: **Felsner, Heldt**
Vorlesung über Graphentheorie/ Graphtheory (DS II) 02. February
http://page.math.tu-berlin.de/~felsner/Lehre/dsII11.html

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(1) Let G be a simple graph with vertex-set $\{1,2,3,4,5,6\}$ and a rotation system

$$\begin{array}{llllll} 1: & 2,6,4,3; & 2: & 1,3,5,6; & 3: & 1,4,5,2; \\ 4: & 1,6,5,3; & 5: & 2,3,4,6; & 6: & 1,2,5,4; \end{array}$$

i.e. vertex 1 has edges to $2,6,5$ and 4 which leave 1 in this cyclic order. Find a planar embedding of G with this rotation system and give a definition of a *face* of a graph with rotation system, that only depends on the rotation system and not on the embedding.

(2) Let H be a simple graph with vertex-set $\{1,2,3,4,5,6\}$ and a rotation system

$$\begin{array}{llllll} 1: & 2,6,3,4; & 2: & 1,5,3,6; & 3: & 1,5,2,4; \\ 4: & 1,3,5,6; & 5: & 2,6,3,4; & 6: & 1,2,5,4; \end{array}$$

Show $H = G$ (with G from (1)). Prove that this rotation system allows a crossing-fee embedding on the torus. Is it possible to an embedding of G on the torus with this rotation system, such that each closed face is homeomorphic to a closed disc?

(3) Let G be planar graph. Prove that the following conditions are equivalent:

- G is bipartite.
- Every face of G has even degree.
- The dual of G is eulerian.

(4) Let G be a planar graph with n vertices such that $G^* = G$. Prove that G has $2(n-1)$ edges. Find such graphs for all $n \geq 4$.