11. Practice sheet for the lecture: Vorlesung über Graphentheorie/ Graphtheory (DS II)

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(1)
(a) Find a 5: 2-coloring of the graph in the picture below.


Prove that there is no 4:2 coloring of this graph.
(b) Find a graph $G$, such that $\chi_{f}(G)>\frac{n}{\alpha(G)}$.
(c) Find a graph $H$, such that $\omega(G)<\chi_{f}(G)<\chi(G)$.
(2) Show that the Kneser graph $K_{t}^{b}$ (with vertex set $\binom{[t]}{b}$ ) has chromatic number

$$
\chi\left(K_{t}^{b}\right) \leq t-2 b+2
$$

(3) Let $G$ be a simple graph. Prove:

$$
\chi(G)=2 \Leftrightarrow \chi_{f}(G)=2
$$

(4) Let $G$ be a vertex-transitive graph with $n$ vertices. Prove:

$$
\chi_{f}(G)=\frac{n}{\alpha(G)}
$$

(5) Let $G$ be a simple graph. Prove that there is an $b \in \mathbb{N}$ such that

$$
\chi_{f}(G)=\frac{\chi_{b}(G)}{b}
$$

