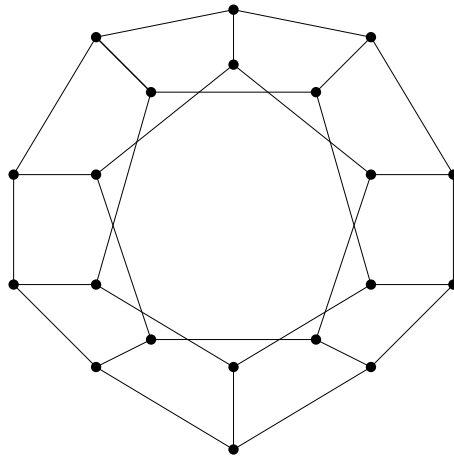


(1)

- (a) Find a 5 : 2-coloring of the graph in the picture below.



Prove that there is no 4 : 2 coloring of this graph.

- (b) Find a graph G , such that $\chi_f(G) > \frac{n}{\alpha(G)}$.
(c) Find a graph H , such that $\omega(G) < \chi_f(G) < \chi(G)$.
- (2) Show that the Kneser graph K_t^b (with vertex set $\binom{[t]}{b}$) has chromatic number

$$\chi(K_t^b) \leq t - 2b + 2.$$

- (3) Let G be a simple graph. Prove:

$$\chi(G) = 2 \Leftrightarrow \chi_f(G) = 2.$$

- (4) Let G be a vertex-transitive graph with n vertices. Prove:

$$\chi_f(G) = \frac{n}{\alpha(G)}.$$

- (5) Let G be a simple graph. Prove that there is an $b \in \mathbb{N}$ such that

$$\chi_f(G) = \frac{\chi_b(G)}{b}.$$