(1)

    (a) What is $\chi(C_5 \,\square\, C_5)$?

    (b) What is $\chi(C_l \,\square\, C_m)$ for all $l, m \in \mathbb{N}$?

    (c) Prove or disprove: Every k-chromatic graph G has a proper k-coloring in which some color class has $\alpha(G)$ vertices.

    (d) Prove or disprove: if G is a connected graph, then $\chi(G) \leq 1 + d_G$ (where d_G is the average degree of G).

(2) Given a set of lines in the plane such that no three of them meet in one point, form a graph G whose vertices are the intersections of the lines, with to vertices adjacent if they appear consecutively on one of the lines. Prove that $\chi(G) \leq 3$.

(3) Let $k \in \mathbb{N} \setminus \{0\}$ and G be a k-regular graph. Prove:

    (a) $V(G)$ odd implies $\chi'(G) > k$.

    (b) $\kappa(G) = 1$ implies $\chi'(G) > k$.

(4) Let G be a graph whose odd cycles are pairwise intersecting, meaning that every two odd cycles in G have a common vertex. Prove that $\chi(G) \leq 5$ and find a graph G with $\chi(G) = 5$ and the condition above.