8. Practice sheet for the lecture: **Felsner, Heldt**
Vorlesung über Graphentheorie/ Graphtheory (DS II) 30. November
http://page.math.tu-berlin.de/~felsner/Lehre/dsII11.html

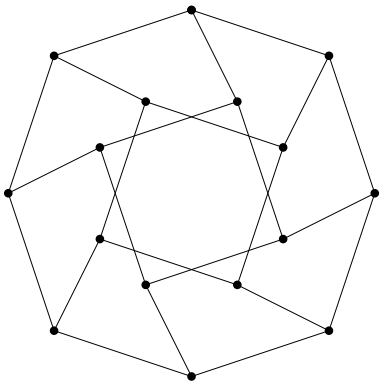
(1)

(a) For wich $n \in \mathbb{N}$ is $K_{n,n}$ Hamiltonian?

(b) For which $n, k \in \mathbb{N}$ is the Turán graph $T_k(n)$ Hamiltonian?

(c) Prove that the Petersen graph is not Hamiltonian.

(d) Let G be the graph in the picture below. Is G Hamiltonian?



(e) For which $d \in \mathbb{N}$ is the Hypercube H_d Hamiltonian?

(2) Let G be a bipartite Hamiltonian graph and x, y vertices of G. Prove that $G - x - y$ has a perfect matching if and only if x and y are on opposite sides of the bipartition of G. Apply this to prove that deleting two unit squares from an 8 by 8 chessboard leaves a board, that can be covered with 1×2 dominos if and only if the two missing squares have opposite colors.

(3) Let G, H be graphs and $G \,\square\, H = (V, E)$ the *box product* of G and H, i.e. the graph with vertices $V = V(G) \times V(H)$ and edges $\{(a, b), (c, d)\} \in E$ for $a, c \in V(G)$ and $b, d \in V(H)$ if either $a = c$ and $\{b, d\} \in E(H)$ or $b = d$ and $\{a, c\} \in E(G)$.

(a) Prove: G, H Hamiltonian $\Rightarrow G \,\square\, H$ Hamiltonian. Can you weaken the requirements?

(b) Prove: G, H vertex transitive $\Rightarrow G \,\square\, H$ vertex transitive.

(4) Let $k \in \mathbb{N}$. Show that there is a tree T_k and a permutation π_k of the vertices of T_k such that the greedy algorithm colors T_k with k colors.