6. Practice sheet for the lecture: **Felsner, Heldt**
Vorlesung über Graphentheorie/ Graphtheory (DS II) 17. November
http://page.math.tu-berlin.de/~felsner/Lehre/dsII11.html

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(1)

(a) Characterize all graphs G which have the property, that all induced subgraphs of G are connected.

(b) Find a graph G which is Eulerian and not Hamiltonian and a graph H, which is Hamiltonian and not Eulerian.

(2)

(a) Prove that the maximum absolute value of an eigenvalue of the adjacency matrix A of a graph G is at most as big as the maximum of the degrees of the vertices of G. To show this, consider general matrices B and the relation of their maximum row-sums and eigenvalues.

(b) Let G be a d-regular bipartite graph, and let A be its adjacency matrix. Prove that $-d$ is an eigenvalue of A and find the corresponding eigenvector. Prove further, that for every eigenvalue μ of A, $-\mu$ is also an eigenvalue. Do you need the regularity of G to prove the second result?

(c) What are the eigenvalues of the adjacency matrix of $K_{m,n}$?

(3)

(a) Prove: The De Bruijn graph $B_n(m)$ contains a set of m edge-disjoint arborescences. Can it contain more?

(b) Prove: The De Bruijn graph $B_n(m)$ is m-edge connected.

(c) Prove: The De Bruijn graph $B_n(m)$ is m-vertex connected.

(4) Let $f : V(B_n(2)) \to V(B_{n-1}(2))$ be the mapping from the vertices of the De Bruijn graph $B_n(2)$ to the Vertices of $B_{n-1}(2)$, which maps $(a_1, \ldots, a_n) \in B_n$ to $(a_1+a_2, a_2+a_3, \ldots, a_{n-1}+a_n) \in B_{n-1}$ with the addition from $\mathbb{F}_2$ (i.e. $1+1=0\ldots$). Let $C = (e_1, \ldots, e_{2^{n-1}})$ be an Euler tour in $B_{n-1}(2)$. Use f to lift C twice to $B_n(2)$, i.e. find $C' = (e'_1, \ldots, e'_{2^{n-1}})$ and $C'' = (e''_1, \ldots, e''_{2^{n-1}})$ in B_n with $f(C') = C = f(C'')$ and use C' and C'' to construct an Euler tour of $B_n(2)$.