5. Practice sheet for the lecture: **Felsner, Heldt**
Vorlesung über Graphentheorie/ Graphtheory (DS II) 10. November
`http://page.math.tu-berlin.de/~felsner/Lehre/dsII11.html`

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(1) Let G be a graph and L the Laplacian matrix of G. Further, let k be the multiplicity of the eigenvalue 0 of L. Prove, that k equals the number of connected components of G.

(2) Prove Cayley's formula using the matrix-tree-theorem.

(3) Let $G_{a,b}$ be the *B-Flower*. Then $G_{a,b}$ consists of a basic cycle $\{v_1, v_2, \ldots, v_a\}$ of length a and a cycles $P_i = \{v_i, w_{i,1}, \ldots, w_{i,b-1}\}$ for $i = 1, \ldots, a$ of length b. So the Vertex-set is

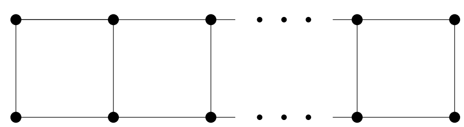
$$V = \{v_1, \ldots, v_a, w_{1,1}, \ldots, w_{1,b-1}, w_{2,1}, \ldots w_{a,1}, \ldots w_{a,b-1}\}$$

and the edges are

$$\begin{aligned} E &= \{\{v_i, v_{i+1}\} \mid i = 1, \ldots, a-1\} \cup \{\{v_1, v_a\} \\ &\cup \left(\bigcup_{i=1}^{a} (\{w_{i,j}, w_{i,j+1}\} \mid j = 1, \ldots, b-2\} \cup \{\{v_i, w_{i,1}\}, \{v_i, w_{i,b-1}\}\}) \right). \end{aligned}$$

Give a picture of $G_{a,b}$ and count the number of spanning trees.

(4) Let G_n be the the graph, which has $2n$ vertices and is shown below.



Let A_n be the adjacency matrix of G_n. Find a recursion formula for $\det(A_n)$.