2. Practice sheet for the lecture: **Felsner, Heldt**
Vorlesung über Graphentheorie/ Graphtheory (DS II) 20. Oktober
`http://page.math.tu-berlin.de/~felsner/Lehre/dsII11.html`

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(1) For which $n \in \mathbb{N}$ is there a Hamilton-path from $(0, 0, \ldots, 0)$ to $(1, 1, \ldots, 1)$ in the boolean lattice $\mathcal{B}_n$?

(2)

(a) Does a 307-regular graph with 911 vertices exist?

(b) Does a 501-regular graph with 10432 vertices exist?

(c) Does a 100-regular graph with 175 vertices exist?

(the interested reader might to try to answer the implicit question *For which pairs* (n, k) *is there a* k*-regular graph with* n *vertices?*, or complete the picture for as many pairs as possible)

(3)

(a) Prove that $(d_1, ..., d_n)$ with $d_i > 0$ for all $i = 1, \ldots, n$ is the degree sequence of a tree if and only if $\sum_{i=1}^{n} d_i = 2n - 2$ holds.

(b) Let $d_1 < d_2 < \ldots < d_k$ be natural numbers. Show that there exists a graph G with $d_k + 1$ vertices, such that the set of degrees in G equals $\{d_1, d_2, \ldots, d_k\}$.

(4)

(a) Let $M = \{a_1, \ldots, a_n\} \subseteq \mathbb{N}$ be a finite set of natural numbers. Let $G_M = (M, E)$ be the (undirected) graph with vertices $a_i \in M$ for $i = 1, \ldots, n$ and edges $\{a_i, a_j\} \in E \subseteq \binom{M}{2}$ for all a_i, a_j such that $a_i | a_j$ or $a_j | a_i$. Is there for every graph G a set M such that $G = G_M$?

(b) Let $M = \{a_1, \ldots, a_n\} \subseteq \mathbb{N}$ be a finite set of natural numbers. Let $H_M = (M, E)$ be the (undirected) graph with vertices $a_i \in M$ for $i = 1, \ldots, n$ and edges $\{a_i, a_j\} \in E \subseteq \binom{M}{2}$ for all a_i, a_j with $gcd(a_i, a_j) = 1$ (gcd = greatest common divisor). Is there for every graph G a set M such that $G = H_M$?