Übungsblatt " Graphs, Order, and Geometry" **SoSe 2024**

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(1)

a. Let G be a comparability graph, show that G contains no odd cycle of length $\geqslant 5$ as induced subgraph.

b. Let G be a incomparability graph, show that G contains no odd cycle of length $\geqslant 5$ as induced subgraph.

(2) Characterize the down-set lattices of orders of width two.

(3) Let G be a graph with a girth $girth(G)$. Show that if $girth(G) > \chi(G)$, then G is a cover graph.

(4) For each m construct a partial order P_m with $\binom{m+1}{2}$ elements such that if $B_1, \ldots, B_k$ is a cover of P_m with the property that each B_i is a chain or an antichain, then $k \geqslant m$. (Later we will see that Greene-Kleitman Theory implies that every order with less than $\binom{m+1}{2}$ elements has such a cover with $k < m$.)

(5) Prove that the following conditions are equivalent:

a. G is a comparability graph of a poset of dimension at most 2;

b. G is a containment graph of intervals on a line;

c. G is a permutation graph.

(6) Let P be a poset and C be a chain in P. Prove that

$$\dim(P) \leqslant \dim(P - C) + 2.$$

(7) Let M be a subset of maximal elements of a poset P. Let $width(P \setminus M) \leqslant w$. Show that

$$\dim(P) \leqslant w + 1.$$