Exercise Sheet 0 in: **Felsner / Schröder**

Discrete Geometry 2 (DG II) 05. April 2018

Due: April 24th

`http://www.math.tu-berlin.de/~felsner/Lehre/DG2-SS18.html`

---

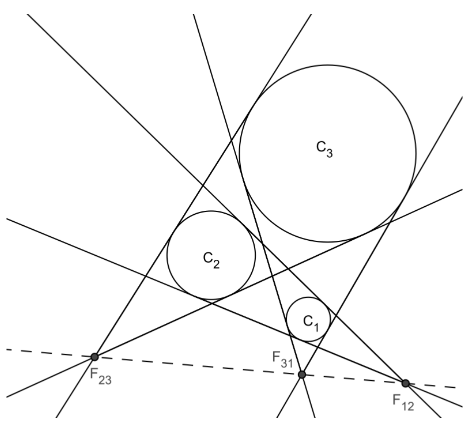
(1) It is, quite obviously, impossible to find 4 points in the plane that have the same distance to each other. But what if 2 distances were allowed? How many ways are there (up to rigid motions) to find a point set $P \in \mathbb{R}^2$ of 4, such that the set of their distances $D = \{|p-q| \mid p, q \in P\}$ has size 2?
Hint: There are only finitely many possibilities.

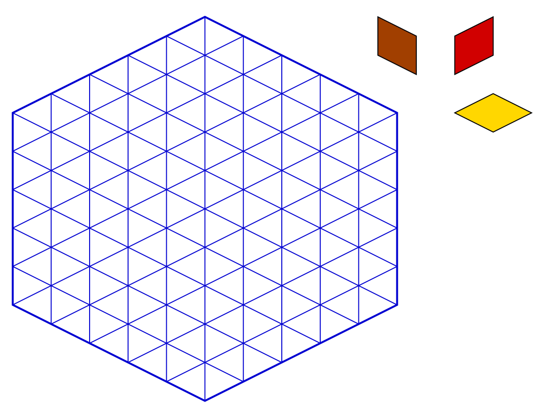
(2) Two circles with different radii in the plane, that do not contain each other, share up to 4 tangents, up to two that separate the circles and two that do not. We will call the intersection point F of the two tangents that do not separate them the *focus point* of the two circles.
Now consider three circles of different radii C_1, C_2, C_3 in the plane, which define 3 focus points, F_{12},F_{23} and F_{31}. We will try to come up with a proof that the focus points always lie on a common line:
Consider an embedding of the plane into $\mathbb{R}^3$ named $P_0 := \mathbb{R}^2 \times \{0\}$. Then define three spheres S_1, S_2, S_3 which have the circles in the plane as their equator and the plane P tangent to the three spheres from above. Then P is non-parallel to P_0 as the spheres have different radii. So the intersection of these two planes is a line.

  (a) Prove that all focus points lie on the common line $P \cap P_0$.

  (b) The proof above is wrong. What is wrong about it?

  (c) Prove the original statement.
  Hint: It is possible to "repair" the proof above.





(3) A regular hexagon can be cut out of a triangular grid. Since the number of triangles in the hexagon is even, it can then be tiled by rhombi consisting of two of these triangles each. These rhombi come in three possible orientations/colors.
Show: Any such tiling consists of an equal number of rhombi of every color.