
Combinatorics (DS I) - Sheet 12

Exercise 12.1.

Which of the following designs exist?

- (a) $S_1(2, 3, 127)$
- (b) $S_2(4, 7, 13)$
- (c) $S_3(2, 10, 25)$
- (d) $S_1(2, 7, 36)$
- (e) $S_{12}(2, 16, 21)$
- (f) $S_3(3, 5, 21)$

Exercise 12.2.

Give a construction of a Steiner-tripple-system $STS(15)$ by considering the edges of K_6 .

Exercise 12.3.

Consider an $n \times n$ chess-board for even n . How many configurations (up to the symmetries of D_4) of n rooks on the board are there, such that no rook can capture another one?

Exercise 12.4.

Inform yourself about the Euler Phi-function φ .

- (a) Show: If $\gcd(a, b) = 1$, then $\varphi(ab) = \varphi(a)\varphi(b)$.
- (b) Compute $\varphi(p^k)$ for a given prime p .

Bonus Exercise

We will call a design a *circle-design*, if $\mathcal{P}$ can be represented as a point set in the plane and $\mathcal{B}$ can be represented as a set of circles in the plane, such that the incidence structure is given by a point lying on a circle. For which values of the parameter t do there exist non-trivial circle designs? Find a set of parameters such that every design with those parameters is a circle-design.

List of hints

- There is either a problem with the some divisibility condition or a construction.
- Consider perfect matchings and pairs of triangles.

- Consider permutation matrixes. Then the elements of the group have nice interpretations in terms of permutations.
- If t is too big, then you can say a lot about the positions of the points.