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**12th Practice sheet for the lecture:  
Combinatorics (DS I)**

**Felsner/ Schröder**  
13th of July 2023

Due date: 18th of July

<http://www.math.tu-berlin.de/~felsner/Lehre/dsI23.html>

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- (1) For which of the parameter sets does a design exist? Either show that there is none or present one. (if  $\lambda$  is omitted, it is 1.)

- (a)  $S(2, 3, 127)$                       (d)  $S(2, 7, 36)$   
(b)  $S_2(4, 7, 13)$                       (e)  $S_{12}(2, 16, 21)$   
(c)  $S_3(2, 10, 25)$                       (f)  $S_3(3, 5, 21)$

[Hint to (f): In the edge set of  $K_7$ , use the cycle  $C_5$ , the star and more as blocks.]

- (\*) We will call a design a *circle-design*, if  $\mathcal{P}$  can be represented as a point set in the plane and  $\mathcal{B}$  can be represented as a set of circles in the plane, such that the incidence structure is given by a point lying on a circle. For which values of the parameter  $t$  do there exist non-trivial circle designs?

- (3) Let  $q$  be a prime power. For every  $k, n \in \mathbb{N}, k \leq n$ , construct the following design:

$$S_\lambda \left( 2, \left[ \begin{array}{c} k \\ 1 \end{array} \right]_q, \left[ \begin{array}{c} n \\ 1 \end{array} \right]_q \right) \text{ with } \lambda = \left[ \begin{array}{c} n-2 \\ k-2 \end{array} \right]_q.$$

- (4) In the lecture we saw two (isomorphic) STS(15).

- (a) Give another construction of an STS(15) by considering the edge set of  $K_6$ , together with edge sets forming triangles and perfect matchings.  
(\*) Show that it is isomorphic to the one from the lecture.