The Exercise session will take place in the room E-N 195 from now on. The room is very close to the room HE 101 connected by two corridors in the 1st floor!

(1) Theorem of Hall

     (a) Find an infinite counterexample to the Theorem of Hall, i.e., find a bipartite graph $G = (X \cup Y; E)$ with the property that $|N(S)| \geq |S|$ for all $S \subset X$ and there is no matching containing all vertices of X.

     (b) The analogue of the Hall condition for general graphs is the *Tutte condition*. Inform yourself about this condition and show at least one implication of Tutte's theorem.

(*) Let $P = (X, \leq)$ be a poset. We call a chain decomposition $\{C_i\}_i$ of P *greedy chain decomposition (GCD)* if it has the following property: C_1 is a maximum chain in P, and for $i > 1$, C_i is a maximum chain in P_i where P_i is the subposet of P induced by $X - \bigcup_{j<i} C_j$.
Prove or disprove: $\exists c \in \mathbb{R}$ such that any GCD of any finite poset P has size at most $c \cdot w$, where w is the width of P.

(3) Consider two magicians M_1, M_2 in well separated rooms. A volunteer picks five cards from a standard deck (52 cards) and hands them to M_1. M_1 keeps one of the five cards and puts the other four (in specific order) in an envelope. The envelope is brought to M_2 who opens it, has a look at the cards and announces the fifth card.

     (a) Explain the existence of a strategy for this trick with the aid of Hall's Theorem.

     (b) Find a playable strategy.

(4) Also prepare the solutions for last week, we will probably have time to discuss some of them in person