
**12. Practice sheet for the lecture:
Combinatorics (DS I)**

Felsner/ Schröder
25. June 2019

Due dates: 2./4. July

<http://www.math.tu-berlin.de/~felsner/Lehre/dsI19.html>

- (1) In the lecture, we deduced the formula $N(\emptyset) = \sum_B (-1)^{|B|} N_{\geq}(B)$. Show that it implies the following *inclusion-exclusion formula*. For $A_1, \dots, A_r \subseteq X$, it holds:

$$\left| X - \bigcup_{i=1}^r A_i \right| = \sum_{I \subseteq [r]} (-1)^{|I|} \left| \bigcap_{i \in I} A_i \right|$$

- (2) How many permutations of the 26 letters of the English alphabet do not contain any of the strings red, blue, or cyan?
- (3) (This exercise gives 2 points.) For a prime power q , consider the poset P of all subspaces of the n -dimensional vector space $V_n(q)$ over \mathbb{F}_q with the subspace relation.
- (a) Show the following identity using one of the q -binomial theorems:

$$\sum_{i=0}^k \begin{bmatrix} k \\ i \end{bmatrix} (-1)^i q^{\binom{i}{2}} = 0$$

- (b) Compute the Möbius function of P .
- (c) Count the number of linear functions from $V_n(q)$ onto $V_k(q)$, that is, the number of surjective linear functions $f : V_n(q) \rightarrow V_k(q)$.
- (4) Use Möbius inversion to show that for every positive integer n , it holds

$$\frac{\phi(n)}{n} = \sum_{d|n} \frac{\mu(d)}{d}$$

[Hint: Recall that $\sum_{d|n} \phi(d) = n$.]