9. Practice sheet for the lecture: Combinatorics (DS I)

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`http://www.math.tu-berlin.de/~felsner/Lehre/dsI13.html`

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(1) Give an example showing that Dilworth's Theorem only holds for finite posets (i.e. find an infinite poset which has only finite antichains but no decomposition into finitely many chains.)

(2)

(a) Show that biregular bipartite graphs $(X \cup Y, E)$ always allow for matchings of size $\min\{|X|, |Y|\}$. A *biregular graph* is a graph where the nodes in X, Y have degree d_x and d_y, respectively.

(b) Show that a regular (i.e. all vertices have the same degree) bipartite graph can be covered with perfect matchings, i.e. that the set of edges can be partitioned into perfect mathchings. Give a lower bound for the number of covers with perfect matchings.

(c) Prove that a graph $G = (V, E)$ is bipartite if and only if there is no odd cycle in G. An odd cycle is a sequence of vertices and edges $v_1, e_1, v_2, ..., e_{2k}, v_n, e_{2k+1}$ such that $e_i + (v_{i-1}, v_i)$ and $e_{2k+1} = (v_{2k+1}, v_1)$ with $e_i \in E$ abd $v_i \in V$.

(4) There is a pair M_1, M_2 of magicians in well separated rooms. A volunteer picks five cards from a standard deck (52 cards) and hands them to M_1. He keeps one of the five cards and puts the other four (in specific order) in an envelope. The envelope is brought to M_2 who opens it, has a look at the cards mumbles and announces the fifth card.
Explain the mathematical idea behind the trick with the aid of Hall's Theorem.

(5) Consider the *stable partnership problem*: Given a graph $G = (V, E)$ modelling the potential partnerships and a preference list for each person $v \in V$. Does there always exist a stable matching?

(6) The analogon of the Hall condition for non-bipartite graphs is the so-called Tutte condition. Inform yourself about this condition and show at least one implication.