4. Practice sheet for the lecture: Combinatorics (DS I)

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`http://www.math.tu-berlin.de/~felsner/Lehre/dsI13.html`

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*Note that the excercises marked with * are the ones of last week.*

(3*)

(a) Show the following identity by a combinatorial argument:

$$S_{n+1,k+1} = \sum_i \binom{n}{i} S_{i,k}$$

(b) The Bell numbers are defined by $\text{Bell}(n) := \sum_k S_{n,k}$. What do they count? Derive a recurrence for them from the identity given in (a) and interpret it combinatorially.

(4*) n persons stand next to each other in a line. Now, they randomly turn either by 90° to the right or left side, such that there may exist pairs which stand face to face. All of these pairs turn by 180° at the same time. Is there a time when nobody turns anymore? If so, after how many time steps do they stop at most?

(5*) In the parliament of some country there are $2n + 1$ seats filled by 3 parties. How many possible distributions (i, j, k), (i.e. party 1 has i, party 2 has j, and party 3 has k seats) are there, such that no party has an absolute majority? Give a nice combinatorial proof.
(Hint: Look at small numbers and make a good guess.)

(6) In how many ways can you pay n Dollar with \$1, \$5, and \$10 bills? Find a generating function and compute the number of ways to pay \$50.

(7) You have three types of stamps, two different types with a value of 2 cent and one type with a value of 3 cent. Now you have to put stamps with a total value of k cent on an envelope. Let h_k be the number of feasible sequences of stamps. Find a closed form for h_k.

(8) Inform yourself about the so called Lagrange Inversion in the context of generating functions. Prepare a 5-minute talk about your findings which also answers the following three questions:

(a) What is the Lagrange Inversion?

(b) State the idea of the proof.

(c) Why is this relevant? (Give examples.)