12. Practice sheet for the lecture: **Felsner/ Kleist, Hoffmann**
Combinatorics (DS I) 25. June '13

Delivery date: 02. – 03 . July

`http://www.math.tu-berlin.de/~felsner/Lehre/dsI13.html`

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(1) Let $v_1, v_2 \in \mathbb{N}$. Show that if a STS(v_1) and a STS(v_2) exist, then there exists a STS($v_1 \cdot v_2$).

(2) Construct a $S_3(3, 5, 21)$. Take the edges as of the K_7 as the vertex set and define the blocks appropriately.

(3) Prove the following equation:

$$\sum_{i=0}^{k} \binom{k}{i} D_{n-i} = \sum_{j=0}^{n-k} (-1)^j \binom{n-k}{j} (n-j)!$$

(4) How many positive integers less than $n \in \mathbb{N}$ have no factor between 2 and 10? How many are these if $n = 1000$?

(5) Let $P = (\mathbb{N}, \leq)$ be the divisor poset of all natural numbers, i.e. $a \leq b$ if and only if $a|b$. Let μ be the Möbius function of P and $r, s \in \mathbb{N}$. Compute $\mu(r, s)$.
You can conclude that for $\frac{r}{s} = a \in \mathbb{N}$ you can define $\mu(a) := \mu(r, s)$ which is the Möbius function from number theory.

(6) Count the number of triples (p_0, p_1, p_2), where p_i are vertex-disjoint lattice paths of length 6 from $A_i = (i, 2 - i)$ to $B_i = (3 + i, 5 - i)$.