9. Practice sheet for the lecture: Combinatorics (DS I) | **Felsner/Heldt, Knauer** 7. June

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`http://www.math.tu-berlin.de/~felsner/Lehre/dsI11.html`

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(1) *Please hand in your solution of this exercise:* How many necklaces of length 12 with beads of 3 colors are there? Count them modulo all symmetries, i.e. rotations and flipping the necklace over.

(2) What is the Cayley graph of a group? Do some literature research to learn about this graph.

(3)

(a) Prove that every group $(G, \circ)$ with an even number of elements contains at least one element $g \neq id$ of order 2, i.e. $g \circ g = id$.

(b) Let $p \in \mathbb{N}$ be a prime number and G a group with p elements. Prove that G is a cyclic group, i.e. there is an element $a \in G$ such that all elements of G are of the form a^k for some $k \in \mathbb{N}$.

(4) Count the number of functions $f : \{0,1\}^3 \to \{0,1\}$ modulo permutations of the variables, i.e. g is equivalent to f if there is $\pi \in S_3$ such that $g(x_1, x_2, x_3) = f(\pi(x_1, x_2, x_3))$.

(5) A graph $G = (V, E)$ is *isomorphic* to a graph $H = (V', E')$, if a re-labeling of the vertices of G equals H, i.e. if there is a bijection $\Phi : V \to V'$ such that the mapping $\Psi((v, w)) := (\Phi(v), \Phi(w))$ is a bijection from E to E'. Furthermore, *loops* are edges, starting and ending in the same vertex.

(a) Count the number of non–isomorphic graphs (without loops) with four vertices.

(b) Count the number of non–isomorphic graphs with four vertices and when loops are allowed.