1. Practice sheet for the lecture: Combinatorics (DS I) **Felsner/ Heldt**

7. April

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`http://www.math.tu-berlin.de/~felsner/Lehre/dsI11.html`

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(1) A spider has a sock and a shoe for each of his eight feet. In how many different ways can he put on his shoes and socks, assuming that on each foot he has to put on the sock first?

(2) Consider a chess tournament of n players, each playing against every other participant. Show that at each point of time during the tournament there exist at least two players, having finished the same number of games.

(3) In the german lottery system "6 aus 49", six pairwise different numbers $a_1 < a_2 < \ldots < a_6$ are drawn from the set $[49] := \{1, 2, \ldots, 49\}$. What is the chance that no two adjacent numbers are picked, i.e. there is no $i \in \{1, 2, \ldots, 5\}$, with $a_i + 1 = a_{i+1}$?

(4) A sequence of numbers $a_1, a_2, \ldots, a_n$ is called *unimodal,* if there exists an $m \in [n]$, such that $a_i \leq a_{i+1}$ for all $i < m$ and $a_i \geq a_{i+1}$ for all $i \geq m$. Give three different proofs of the unimodality of the sequence $\binom{n}{1}, \binom{n}{2}, \binom{n}{3}, \ldots, \binom{n}{n}$ for all $n \in \mathbb{N}$, based on the three given hints:

(a) Use the definition
$$\binom{n}{k} := \frac{n!}{k! \cdot (n-k)!}.$$

(b) Consider the recursive definition
$$\binom{n}{k} := \binom{n-1}{k-1} + \binom{n-1}{k} \text{ and } \binom{n}{0} = \binom{n}{n} = 1,$$
based on Pascale's triangle.

(c) Use the bijection between $\binom{n}{k}$ and the number of subsets of $[n]$, having k elements.

(5) There are nine caves in a forest, each being the home of one animal. Between any to caves there is a path, but they do not intersect. Since it is election time, the candidating animals travel through the forest to advertise themselves. Each candidate visits every cave, starting from their one cave and arriving there at the end of the tour again. Also each path is only used once in total, since nobody wants to look at the embarrassing posters (not even their own), put there on the first traverse. How many candidates are there at most?