8. Practice sheet for the lecture: **Felsner/ Tiwary, Heldt**
Combinatorics (DS I) 12. June

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`http://www.math.tu-berlin.de/~felsner/Lehre/dsI09.html`

(1)

(a) Take a standard deck of 52 playing cards. Split them into 13 piles S_i, each containing 4 cards. Show that for any such splitting you can choose one card $a_i \in S_i$ from each set, such that the set $\{a_1, \ldots, a_{13}\}$ contains one card of each of the ranks $\{2, 3, 4, \ldots, 10, \text{jack}, \text{queen}, \text{king}, \text{ace}\}$.

(b) Show that every regular, bipartite Graph has a perfect matching. A bipartite graph is called *regular* if every vertex has degree d and a matching is a *perfect matching* if touches every vertex.

(2) Give an example, showing that Dillworth's theorem only holds for finite posets (i.e. find an infinite poset with finite antichains but no decomposition into finitely many chains).

(3) Let G be a graph and M be a matching of G. Color all edges e of G blue if $e \in M$ and red otherwise. The vertex v is *exposed* if all adjacent edges are red (i.e. do not belong to the matching). Furthermore a path between two vertices is *alternating colored,* if the path's edges are alternating red and blue. Show, that a matching is maximum (i.e. there is no matching, containing more edges) if and only if for all pairs of exposed vertices v, w there is no alternating path.

(4) A member of the staff of a *jobcenter* has to assign jobs to unemployed persons. Today there are 17 jobs and 20 persons to match, but not everybody qualifies for every job (there are 126 pairs (person, job), meeting the requirements). There are five unemployed engineers and seven engineering jobs. Only the engineers qualify for this jobs and there are only 16 valid pairs (engineer, engineering job). Next there are five jobs everybody, except the engineers, would accept. The last five jobs require one of eight qualified craftsman. Give tight upper and lower bounds for the number of jobs which can be assigned to persons, meeting the qualification requirements.