**7. Practice sheet for the lecture: Combinatorics (DS I)** — **Felsner/ Tiwary, Heldt**

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`http://www.math.tu-berlin.de/~felsner/Lehre/dsI09.html`

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(1) In an office, at various times during the day, the boss gives the secretary a letter to type, each time putting the letter on top of the pile in the secretary's inbox. When there is time, the secretary takes the top letter of the pile and types it. There are nine letters to be typed during the day, and the boss delivers them in order 1,2,3,4,5,6,7,8,9. While leaving for lunch, the secretary tells a colleague that letter 8 has already been typed, but says nothing else about the morning's typing. The colleague wonders which of the nine letters remain to be typed after lunch and in what order they will be typed. Based upon above information, how many such *after lunch typing orders* are possible? (That there are no letters left to be typed is one of the possibilities.)

(2) For any poset $(P, \leq)$ let $\max((P, \leq)) := \{x \in P \mid x \leq y \Rightarrow y = x\}$ be the set of maxima of $(P, \leq)$ and $e((P, \leq))$ be the number of linear extensions of $(P, \leq)$. Prove

$$e((P, \leq)) = \sum_{x \in \max(P)} e((P \setminus \{x\}, \leq')),$$

where $\leq' := \leq \cap (P \setminus \{x\}) \times (P \setminus \{x\}) \subseteq P \times P$ is the restriction of $\leq$ to $P \setminus \{x\}$.

(3)

(a) Let $(P, \leq)$ be a poset, consisting of $n$ disjoint chains of length $a_1, a_2, \ldots, a_n$. How many linear extensions does $P$ have?

(b) Solve again exercise 1 of sheet 1. Use part (a) of this exercise to do so. The corresponding exercise was: *A spider has a sock and a shoe for each of his eight feet. In how many different ways can he put on his shoes and socks, assuming that on each foot he has to put on the sock first?*

(4) Count recursively the number of linear extensions of *tree–shaped posets,* i.e. posets $(P, \leq)$ such that for each $x \in P$ there is a parent $y \in P$ such that for all $z \in P$ with $z \leq x$ we have $z \leq y \leq x$ (hint: consider and use $h(x) := \#\{z \in P : x \leq z\}$).

(5) Review exercise 1 of this sheet.

(a) Model the typing sessions as a poset (each element in the poset is a possible status, where some letters were handed in and some were typed). Draw this poset for the case of three letters. Does this poset have a rank function? Does it have a symmetric chain decomposition?

(b) Give an upper bound of the total number of typing sessions by applying exercise 4.