4. Practice sheet for the lecture: Combinatorics (DS I)

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`http://www.math.tu-berlin.de/~felsner/Lehre/dsI09.html`

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(1) Show that there is a formal power series $f(x) = \sum_{i=0}^{\infty} a_i x^i$, such that $f(x) = \sqrt{1+x}$ and compute the coefficients $a_0, \ldots, a_5$.

(2) You have three types of stamps, two different types with a value of 2 cent and one type with a value of 3 cent. Now you have to put stamps with a total value of k cent on an envelope. Let h_k be the number of feasible sequences of stamps. Find a closed form for h_k.

(3) Let $(f)_n \in \mathbb{R}^{\mathbb{N}}$ be a sequence and $a_1, \ldots, a_k, b_1, \ldots, b_k \in \mathbb{R}$ some scalars. Consider the linear rekursion

$$f_n = a_1 f_{n-1} + a_2 f_{n-2} + a_k f_{n-k} \text{ for all } n > k$$

with initial conditions $f_1 = b_1, f_2 = b_2, \ldots, f_k = b_k$. As shown in the lecture, we can represent

$$F(x) = \sum_{i=0}^{\infty} f_n x^n = \frac{Q(x)}{P(x)}$$

with polynomials $Q(x), P(x) \in \mathbb{R}[x]$. Compute the coefficients of $Q(x)$ in terms of a_i and b_j.

(4) In how many ways can you pay n Dollar with 1\$,5\$ and 10\$ notes? Find a generating function and compute the number of ways to pay 50 Dollar.

(5) Proof

$$\begin{bmatrix} n \\ m \end{bmatrix}_q \begin{bmatrix} m \\ k \end{bmatrix}_q = \begin{bmatrix} n \\ k \end{bmatrix}_q \begin{bmatrix} n-k \\ m-k \end{bmatrix}_q$$

for all $n \geq m \geq k \geq 0$ (Try to give two or three different proofs).

(6) Show

$$\sum_{i=0}^{n} \begin{bmatrix} i \\ k \end{bmatrix}_q \cdot q^{(k+1)(n-i)} = \begin{bmatrix} n+1 \\ k+1 \end{bmatrix}_q$$

(Try to give two or three different proofs).