Projekt C19*

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Analysis and numerics of the peridynamic equation

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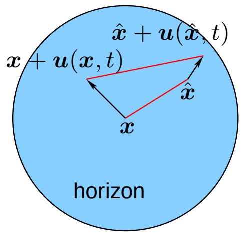
Introduction

The peridynamic model is a non-local theory in continuum mechanics that avoids spatial derivatives. The equation of motion reads as

$$\rho(\boldsymbol{x})\partial_t^2\boldsymbol{u}(\boldsymbol{x},t) = \int_{\mathcal{V}} \boldsymbol{f}_{d,\delta}(\boldsymbol{x},\hat{\boldsymbol{x}},\boldsymbol{u}(\boldsymbol{x},t),\boldsymbol{u}(\hat{\boldsymbol{x}},t),t)\,d\hat{\boldsymbol{x}} + \boldsymbol{b}(\boldsymbol{x},t)\,, \quad (\boldsymbol{x},t)\in\mathcal{V}\times(0,T), \mathcal{V}\subseteq\mathbb{R}^d.$$



It essentially relies upon differences of the displacement of material points interacting within a prescribed horizon of radius δ. Here also lies the possible advantage as the evolution of discontinuities in the displacement might be inherently described within the peridynamic model. Typical applications are the autonomous propagation of cracks.

Model

One of the simplest **nonlinear** models based on the idea of a spring uses the ansatz

$$\boldsymbol{f}_{d,\delta} = \begin{cases} c_{d,\delta}\, s\, \boldsymbol{e} & \text{if } \|\hat{\boldsymbol{x}}-\boldsymbol{x}\| < \delta \\ 0 & \text{else} \end{cases} \quad \text{with}$$

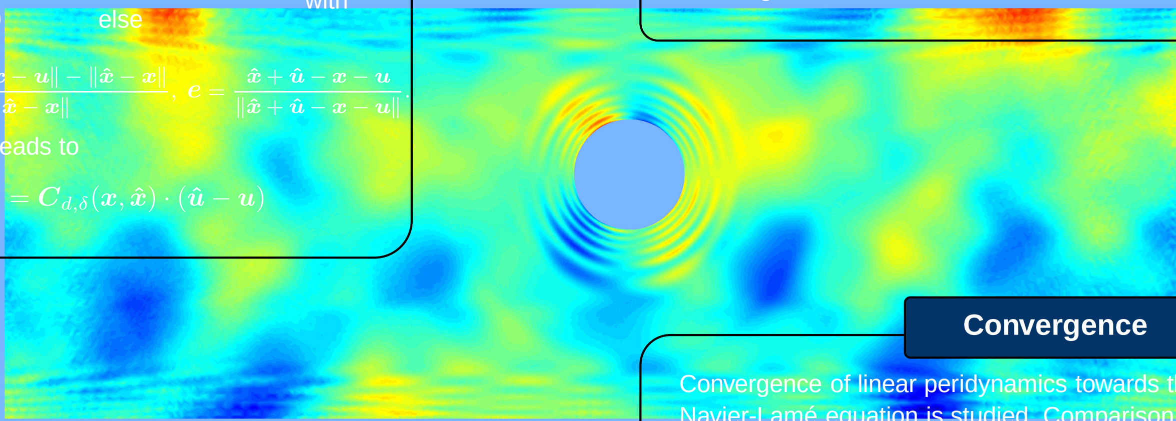
$$s = \frac{\|\hat{\boldsymbol{x}}+\hat{\boldsymbol{u}}-\boldsymbol{x}-\boldsymbol{u}\| - \|\hat{\boldsymbol{x}}-\boldsymbol{x}\|}{\|\hat{\boldsymbol{x}}-\boldsymbol{x}\|}, \quad \boldsymbol{e} = \frac{\hat{\boldsymbol{x}}+\hat{\boldsymbol{u}}-\boldsymbol{x}-\boldsymbol{u}}{\|\hat{\boldsymbol{x}}+\hat{\boldsymbol{u}}-\boldsymbol{x}-\boldsymbol{u}\|}.$$

Linearisation leads to

$$\boldsymbol{f}_{d,\delta} = \boldsymbol{C}_{d,\delta}(\boldsymbol{x},\hat{\boldsymbol{x}})\cdot(\hat{\boldsymbol{u}}-\boldsymbol{u})$$

Well-Posedness

Existence and uniqueness is shown for the linear case but is open for the nonlinear case. Stability w.r.t. initial data, right hand side $\boldsymbol{b}$, and the integral kernel is also studied.



Convergence

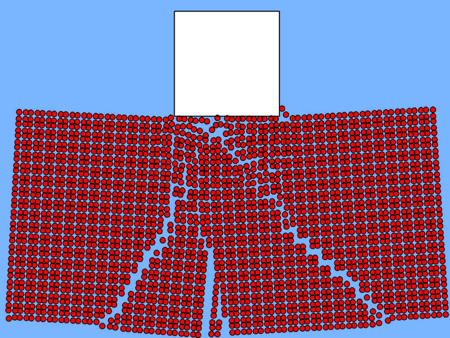
Convergence of linear peridynamics towards the Navier-Lamé equation is studied. Comparison with the classical theory needs to identify

- material constants
- the solution's behaviour at the boundary

Different force ansatzes and dispersion relations correspond to different mechanical models.

Spatial approximation

For the numerical solution, the quadrature formula method with different types of Gauß- and Newton-Cotes quadratures on triangles and quadrilaterals is used. Numerical simulations confirm expected convergence rates.



Cooperation

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