Qualitative properties of reaction-diffusion equations

Yannick Sire (Marseille)

June 9, June 16, June 23
at 4:15 pm in MA 544

The aim of the course is to present results on some non linear elliptic equations arising in combustion theory. I will develop the following aspects.

1. Reaction-diffusion equations. Travelling wave solutions. Existence theory for travelling waves in the $1D$ framework.

2. Theory of uniformly elliptic equations: existence, regularity, Schauder theory, Agmon-Douglis-Nirenberg theory, Harnack estimates, boundary Harnack principle. Elements of the theory of parabolic equations (semi-group theory).

3. Existence of multi-dimensional travelling waves. Proof of the Bérestycki-Larrouturou-Lions theorem. I will present a proof using a continuation argument and a proof using topological methods, such as the Leray-Schauder degree.

4. Linear and non linear stability of the travelling waves.

5. Free boundary problems and high energy activation limits. Non degeneracy near the free boundary, optimal regularity, free boundary condition, Hausdorff estimates (after Bérestycki, Caffarelli and Nirenberg).

If time permits, I will develop several topics related to research among: homogenization, travelling waves in periodic media, anomalous diffusion, theory of free boundaries, regularity of flat level sets (De Giorgi conjecture).

Seminar talk within the Colloquium of the Arbeitsgruppe
Modellierung • Numerik • Differentialgleichungen
on June 17 at 4:15 pm in MA 313.

Multiscale methods coupling atomistic and continuum mechanics

Frédéric Legoll (Marne-la-Vallée)

June 30, July 7, July 14
at 4:15 pm in MA 544

In order to describe a solid which deforms smoothly in some region, but non smoothly in some other region, many multiscale methods have been recently proposed, that aim at coupling a particle model (discrete mechanics) with a macroscopic model (continuum mechanics). Applications include fracture propagation in metals, and nanoindentation. One such multiscale method is the Quasi-Continuum method (Tadmor et al 1996, Miller and Tadmor 2002).

After reviewing some of these methods, we will present a theoretical analysis for such a coupling in a one-dimensional setting. We study both the general case of a convex energy and a specific example of a nonconvex energy, the Lennard-Jones case, and obtain error estimators. In the latter situation, we will see that the coupled model needs to account in an adequate way for the coexistence of a discrete model and a continuous one. Otherwise, spurious effects may appear.

The extension of the method to the finite temperature case will next be discussed. The problem then amounts to computing statistical mechanics averages of observables that depend on only a few atoms of the system. In this context of coarse-grained models of materials, another interesting quantity is the free energy, which is indeed related to the constitutive relation of the material, at a given temperature. Our approach is based on a thermodynamic limit procedure, and makes use of ergodic type theorems and large deviation theory. For one-dimensional models, it provides a possible efficient computational strategy.

Seminar talk within the Colloquium of the Arbeitsgruppe
Modellierung • Numerik • Differentialgleichungen
on July 8 at 4:15 pm in MA 313.

Technische Universität Berlin

Lecture series on

Analytical and numerical aspects of partial differential equations

During the summer semester 2008, we organize a series of lectures on different aspects of partial differential equations held by young mathematicians from France. The lectures address the following topics:

- Adaptive semi-lagrangian schemes for the Vlasov-Poisson equation (Dr. Martin Campos Pinto, Strasbourg)
- Coupling of scalar conservation laws (Dr. Julien Jimenez, Pau)
- Hyperbolic conservation laws (Dr. Boris Andreianov, Besançon)
- Qualitative properties of reaction-diffusion equations (Dr. Yannick Sire, Marseille)
- Multiscale methods coupling atomistic and continuum mechanics (Dr. Frédéric Legoll, Marne-la-Vallée)

Knowledge in partial differential equations (as in *Differentialgleichungen II*) and basic knowledge in numerical analysis (as in *Einführung in die Numerische Mathematik*) are prerequisites. An introduction is given on April 14, 2008 at 4:15 pm in MA 544.

For more information, please visit:
`www.math.tu-berlin.de/~emmrich/luft.htm`

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Prof. Dr. Petra Wittbold
Priv.-Doz. Dr. Etienne Emmrich

Institut für Mathematik
AG Modellierung • Numerik • Differentialgleichungen
Sekretariat MA 6-4

Adaptive semi-lagrangian schemes for the Vlasov-Poisson equation

Martin Campos Pinto (Strasbourg)

April 21, April 28 at 4:15 pm in MA 544
May 14 at 4:15 pm in MA 313

In this lecture, I will describe a few novel adaptive schemes for solving the Vlasov-Poisson equation

$$\partial_t f(t,x,v) + v \cdot \partial_x f(t,x,v) + E(t,x) \cdot \partial_v f(t,x,v) = 0,$$

$$\partial_x E(t,x) = \int f(t,x,v)\,\mathrm{d}v - 1, \quad f(0,\cdot,\cdot) = f_0 \in \mathcal{C}(\mathbb{R}^2),$$

which describes in statistical terms the nonlinear evolution of a collisionless plasma.

In order to save computational costs while approximating the complex and thin structures that may appear in the solutions, several adaptive schemes have been proposed in the past few years. A common feature of these schemes lies in the hierarchical discretization (e.g. wavelets or adaptive finite elements) of the phase space and it is a critical issue to generate adaptive meshes that are optimal in the sense that they only retain the "necessary" grid points.

Based on the regularity analysis of the numerical solution and how it gets transported by the numerical flow, I will show that it is possible to perform an accurate evolution of the adaptive mesh from one time step to the next one, in the sense that the accuracy of the scheme is monitored by a prescribed tolerance parameter ε which represents the local interpolation error at each time step. As a consequence, the numerical solutions can be proved to converge towards the exact ones as ε and Δt tend to zero, provided the initial data has enough (Sobolev) smoothness. In addition to this error analysis, I shall give some theoretical arguments concerning the optimality of the adaptive meshes, as well as numerical tests that illustrate their effectiveness.

Seminar talk within the Colloquium of the Arbeitsgruppe
Modellierung • Numerik • Differentialgleichungen
on May 13 at 4:15 pm in MA 313.

Coupling of scalar conservation laws

Julien Jimenez (Pau)

May 5 at 4:15 pm in MA 544

This lecture is devoted to the mathematical analysis of a coupling problem between a purely quasilinear convection equation set in a hyperbolic area and a diffusive one, set in a parabolic domain complementary to the former.

When we deal with hyperbolic equations, it is known that we are interested in the existence and uniqueness property of a particular weak solution, called weak entropy solution. Here we define a notion of weak entropy solution adapted to the coupling problem we consider and then we prove that this solution exists and is unique.

To do so we combine some classical methods used in the theory of nonlinear parabolic equations (namely the Schauder-Tychonoff fixed point theorem and the Holmgren-type duality method) and some techniques linked to hyperbolic problems such as the method of doubling variables and the vanishing viscosity method.

Seminar talk within the Colloquium of the Arbeitsgruppe
Modellierung • Numerik • Differentialgleichungen
on May 7 at 4:15 pm in MA 313.

Hyperbolic conservation laws

Boris Andreianov (Besançon)

May 19, May 26, June 2
at 4:15 pm in MA 544

The course is based upon the lecture notes of S.N. Kruzhkov, the founder of the modern theory of hyperbolic conservation laws. This is a survey introduction to the basic theory of the one-dimensional scalar conservation law $u_t + f(u)_x = 0$. A part of the course will be given under the form of exercises. The tentative programm includes:

- Examples: the transport equation,
  the Burgers equation, the traffic flow.
- Classical solutions. The method of characteristics.
  Formation of singularities. Condensation waves.
- Weak solutions. Rankine-Hugoniot conditions.
  Shock and rarefaction waves. Time reversibility.
- Examples of non-uniqueness of weak solutions.
  The Oleinik and Lax admissibility conditions.
- The vanishing viscosity approach. Travelling waves.
  Admissible and non-admissible shocks. Irreversibility.
- The entropy inequality. Kruzhkov entropy solutions.
- Uniqueness of piecewise regular and BV solutions.
- The Riemann problem. Self-similar solutions.
- Construction of approximate solutions. Viscosity.
  Wave-front tracking. Glimm and Godunov schemes.
- Existence of entropy solutions.
- Multi-D conservation laws, boundary-value problems,
  systems of conservation laws: what's known ?

The relevant references are the introductory chapters of the books by A. Bressan (Oxford University Press, 2000) and by H. Holden and N.H. Risebro (Springer, 2002).

Seminar talk within the Colloquium of the Arbeitsgruppe
Modellierung • Numerik • Differentialgleichungen
on May 27 at 4:15 pm in MA 313.