Modelling unbalanced pedestrian flow on a two-dimensional grid

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Abstract: To some extent, pedestrian flow can be considered as a compressible fluid. However, since any real pedestrian would claim a minimum physical space, pedestrian flow becomes incompressible once the density reaches a threshold value. The size of this minimum physical space may even vary under circumstances of different social contexts. In the current paper, we discuss the possibility of describing this phenomenon on a two-dimensional grid. On this grid, the notion of "virtual grid cell" is applied to describe the aforesaid minimum space. The basic idea of our model is applicable for the general grid-based methods of pedestrian flow simulation.

Keywords: cellular automaton, Lighthill-Whitham model, pedestrian flow, variable cell size

1 Introduction

The concept of cellular automaton (CA) is an important modelling technique of pedestrian flow simulation [see 12, 13], [and 2, 6, 7, specifically]. The CA models are generally categorized to be microscopic, that is, the pedestrians (called *particle*s) are investigated individually. The overall behaviour of the particles in such a simulation system is described by the cell state change of the CA, which is further defined and regulated by the automaton's transition rules. Since the cell state is always well-defined by the transition rules, it is obvious that at every position represented by the cells of the automaton geometrically, there can be only two possibilities: either, exactly one particle is currently located at this position, or no particle is at this position. Therefore, we can understand the automaton cell as the minimum space a particle reserves for itself exclusively. Common configurations [e. g. 2, 6] set the automaton cell size to be 0.4m × 0.4m. This implies that the distance between two particles can be 0.4m in normal cases. These models addressed the problem of $v_{\max} = 1$[1] in which particles are allowed exclusively a position change to the next adjacent cell in a simulation clock. However, [11] pointed out that a minimum width of 0.6m was necessary for undisturbed movement. From empirical observations we wish to point out that a preferred distance from others would be approximately 0.7–1.0m, when a real pedestrian is not walking in a group with others.

In our previous work [3], the grid cell size was configured to be 0.5m × 0.5m, and the simulation clock length was set to be $\Delta t = 1$s. A step size of 3 (grid cells in one dimension) in each simulation cycle thus corresponded to a walking speed of $1.5\mathrm{m\,s}^{-1}$, conforming to both the statistics collected by [10] and empirical observations. The multiple-cell-step (i. e. $v_{\max} > 1$) was computed on the grid using J. E. Bresenham's algorithm of line rastering. Considering the fact that real pedestrians claim a relatively larger space, [1] suggested a possibility of defining virtual variable cell size for the particles. The so-called `d/a`-mechanism enabled a particle to deactivate the surrounding grid cells to be occupied or entered by other particles. As an effect of this, the minimum exclusive space claimed by the particles (i. e. the individual pedestrians) can be modified in a virtual way.

1. This is the common simplified notation for $v_{\max} \cdot \Delta t = 1$ automaton cell size in one dimension, with Δt being the simulation clock time span.

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In the current paper, we consider a simulation system of pedestrian dynamics in which the sink (flow-out) is not equal to the source (flow-in). With the unbalancing source and sink, the particle density varies. The particle density has a twofold influence in the simulation: it is related to the inverse of the minimum space which the particles claim and at the same time, it decides the local particle velocities. We investigate the pedestrian flow in this context. To make our model independent of other factors, we neglect the more specific interactions among the particles [see 3], the particles are given a destination and they form a flow on a global basis accordingly.

2 Model

2.1 Grid construction

[1] mentioned the fact that real pedestrians claim a relatively large space in normal situations. Beyond that, pedestrians normally do not exhibit any influence on others. In other cases (e. g. emergency), the size of this exclusive space claimed decreases drastically till a certain level is reached. Thus, pedestrian flow is "compressible" to certain extent. [1] considered two extreme cases in which a particle (i. e. a real pedestrian) reserves a space of 0.9m × 0.9m and 0.3m × 0.3m respectively. The former resembles the state that a real pedestrian walks in leisure; the latter is the actual size of a particle and cannot be compressed further. The advantage of this configuration is that in the first case a particle takes nine times the planar space as in the second case and can be easily implemented on a regular grid.[2]

This enables us to construct the underlying grid with a cell size of 0.3m × 0.3m for the simulation; and a planar space of 0.9m × 0.9m refers to a set of 3 × 3 grid cells. We consider two states of the eight surrounding grid cells (with a particle's current position as the origin): *activated* (abbreviated: `a`) or *deactivated* (abbreviated: `d`). In order to admit the entrance to a position, we request that the associated grid cell be activated for the particles. Figure 1 (left and right) describes the grid cell states for the aforesaid two extreme cases. In addition, it is possible that the surrounding cells are only partially activated (cf. Figure 1 middle).

Furthermore, we may describe the grid cell state of `a` or `d` by assigning a probability of deactivation p, see Figure 2.[3] This probability p is further dependent on an index variable $\imath$: $\imath = 1$ denotes the normal case mentioned above in which a particle claims nine grid

d	d	d
d		d
d	d	d

a	d	a
d		d
a	d	a

a	a	a
a		a
a	a	a

Figure 1: Activating and deactivating grid cells.

cells whereas $\imath = 0$ denotes the other case in which only exactly one cell is requested.

1	1	1
1	1	1
1	1	1

$\imath = 1$

p	1	p
1	1	1
p	1	p

0	1	0
1	1	1
0	1	0

$\imath = \frac{1}{2}$

0	p	0
p	1	p
0	p	0

0	0	0
0	1	0
0	0	0

$\imath = 0$

Figure 2: Associating grid cells with a probability of being deactivated.

In [1], possible constructions of the function p were discussed. We now investigate the particle density. A straight way to express this is the mass of the particles per volume. After adapting it to the two-dimensional grid and re-scaling both the particle volume and the grid size, this becomes

$$\rho = \frac{\text{number of the particles}}{\text{number of the grid cells}}.$$

In this way, the density is computed in two dimensions. Similarly, this density can be calculated on a local basis. Since the current model focuses on the simplest situation in which the particles are given a fixed destination, the position change of the particles in the second dimension (with the first being the flow direction) should be less interesting than in the first dimension. Thus, it suffices us to consider the density in one dimension. In the sequel, density will be computed columnwise on a two-dimensional grid.

In our model of simulation, particles will be produced on a pre-configured basis in the leftmost column of the grid. We call these "flow-in". (Similarly, particles leaving the rightmost column of the grid will be called "flow-out".) In Figure 3, the flow-in rates (in comparison to the size of column, i. e. the number of grid cells in one column) of the particles can be considered as the particle densities in the first column of the two-dimensional grid. In the notation of [4],

2. For an irregular grid, it may also be implemented, once an adequate notion of neighbourhood can be defined on this grid.

3. The cells at the origin are deactivated, because they represent the current position of the particles and entrance into it by other particles should be rejected.

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the columns represent the second dimension of the particle flow, i. e. the deviation from the local flow direction.[4]

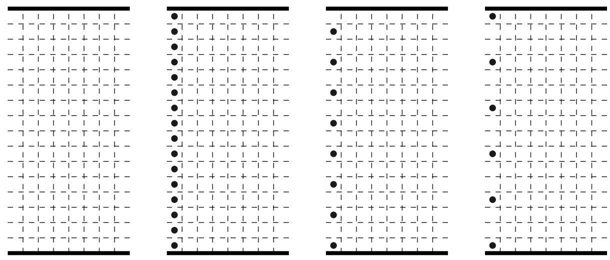

Figure 3: Different flow-in densities: 0, 1, $\frac{1}{2}$ and $\frac{1}{3}$. The particles flow from the left to the right.

In Figure 3, the particle density in the first column of the grid is 0, 1, $\frac{1}{2}$ and $\frac{1}{3}$ respectively. Among these, 1 and $\frac{1}{2}$ present two critical cases. 1 implies that a particle takes exactly one grid cell, and refers therefore to the threshold of the particles' compressibility and indicates $\imath = 0$. $\frac{1}{2}$ implies that a nontrivial (i. e. not on the boundary etc.) particle is positioned in a grid cell with two further empty grid cells on both sides. Therefore, for a density ρ, the indicating index $\imath$ for the deactivation probability p can be defined in the following way,

$$\imath : [0,1] \to [0,1],$$
$$\imath = \begin{cases} 1, & \text{if } 0 \le \rho \le \frac{1}{2}, \\ 2(1-\rho), & \text{if } \frac{1}{2} \le \rho \le 1, \\ 0, & \text{if } \rho = 1. \end{cases}$$

[1] proposed a deactivation probability p[5] defined by

$$p : \left[0, \tfrac{1}{2}\right) \cup \left(\tfrac{1}{2}, 1\right] \to [0,1],$$
$$p = \begin{cases} 2\imath, & \text{if } 0 \le \imath < \frac{1}{2}, \\ 2\imath - 1, & \text{if } \frac{1}{2} < \imath \le 1, \end{cases}$$

depending on the index $\imath$. (p will not be needed for $\imath = \frac{1}{2}$, see Figure 2.)

The size of the second dimension of the grid (i. e. the number of grid cells in the column) can be considered as the flow capacity of the system. The capacities of flow-in and flow-out (both larger than 0) play a significant role. We define

$$\alpha = \frac{\text{number of grid cells in the first column}}{\text{number of grid cells in the last column}} \tag{1}$$

to denote the ratio of the flow-in and flow-out capacities. Without loss of generality, we assume $\alpha \ge 1$ (the case $\alpha \le 1$ can be interpreted in full analogy). Let ρ_{in} and ρ_{out} denote the flow-in and flow-out particle density respectively. Let $\imath_{\text{in}}$ and $\imath_{\text{out}}$ denote the indicating index variable for deactivation probability for flow-in and flow-out respectively. For $\alpha\rho_{\text{in}} > 1$, particle congestion would be unavoidable, because

$$\alpha\rho_{\text{in}} = \frac{\text{flow-in capacity}}{\text{flow-out capacity}} \cdot \rho_{\text{in}},$$

and therefore,

$$\text{flow-in capacity } \cdot \rho_{\text{in}} > \text{flow-out capacity} \cdot 1.$$

To achieve a balanced flow, ρ_{out} must be larger than 1, which is not possible. In the test cases described later, we will see this confirmed.

2.2 Velocity and step calculation on the grid

In the research of vehicular traffic flow, the correlation of the density of the vehicles and their average velocities had been discovered at an early stage [5, 8] and adapted in models like [9]. This phenomenon can be described by

$$v(\rho) = v_0 \left(1 - \frac{\rho}{\rho_{\max}}\right).$$

This yields

$$\frac{\partial v}{\partial \rho} = -\frac{v_0}{\rho_{\max}},$$

and gives us an approximation of the velocity change in particular in the vicinity of $\rho = 0$. Overall, the velocity v tends to take the undisturbed value v_0 when the density ρ is low, and for high densities, the velocity v decreases to zero due to congestions.

On a two-dimensional grid, $\rho_{\max}$ can be simply set to be 1, since in each of the grid cell of the underlying geometry there can be at most one particle present. For the individual particles, since its moving speed is dependent of the environmental settings, ρ should be evaluated locally.

We apply the basic configuration of [3] for $\Delta t = 1\text{s}$ as the time length of each simulation step, $v_0 = 1.5\,\text{m}\,\text{s}^{-1}$ as the undisturbed average velocity of the

4. The overall flow direction needs not to be a straight line, this is, the underlying two-dimensional grid may not be regular.

5. [1] noted that different definitions are admittable without showing significantly different effects as long as the bounding condition $p(0) = 0$, $p(\frac{1}{2}-) = 1$, $p(\frac{1}{2}+) = 0$, $p(1) = 1$ is met and the p is monotonic in the two sub-intervals.

pedestrians. Given the grid cell of size of 0.3m in one dimension, a normal, undisturbed step would correspond to five cells on the grid. We refer to [3] for the step calculation for multiple cells using J. E. Bresenham's algorithm. The step computed in this way stands for the shortest way from one position to another on the two-dimensional grid. To make the model independent, we introduce no further step calculation schemes. In other words, the particles merely "flow" from one end (the source) to the other (the sink) of the simulation system.

3 Notes on the Implementation

This section may serve to give a correct implementation. To this purpose, three aspects need to be investigated closely.

3.1 Information and structure

We consider a position indexed as (i, j) on the two-dimensional grid. A particle at this position affects up to eight[6] other cells positioned at

$$\begin{array}{ccc} (i-1, j+1), & (i, j+1), & (i+1, j+1), \\ (i-1, j), & & (i+1, j), \\ (i-1, j-1), & (i, j-1), & (i+1, j-1), \end{array} \quad (2)$$

in terms of deactivation probability. Indices undefined[7] on the grid should be neglected.

Consequently, the state of a grid cell may be influenced by up to eight particles in the surrounding cells. For a single grid cell, this information can be retrieved by investigating its immediate neighbours, for a to-be-deactivated grid cell, it is important to note by whom[8] it is to be deactivated and the relevant deactivation probability value, cf. Figure 4.

For a grid cell under multiple influence of deactivation, there are two possible interpretations of the deactivation probability. Suppose a grid cell is to be deactivated by its neighbour N_1 with a probability p_1 and a second neighbour N_2 with p_2. When we take these two events as independent, the probability for a deactivation for this grid cell would be

$$1 - (1 - p_1)(1 - p_2) = p_1 + p_2 - p_1 p_2,$$

since it must not be deactivated by N_1 and at the same time, not by N_2, either. On the other hand, due

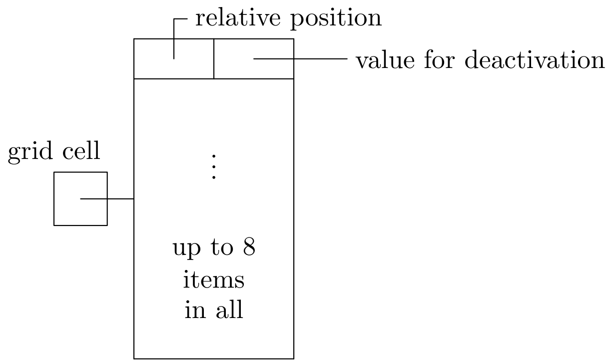

Figure 4: Saving all the deactivation information for a grid cell in a container holding up to eight items. Each item contains a pointer to the position of the affecting particle and the relevant value of deactivation probability.

to the heuristic nature of the construction of p (see [1] for details), the alternative way

$$\max (p_1, p_2)$$

offers an acceptable approximation. Referring to Figure 4, this means that the largest value among all items plays a key role for the grid cell investigated. Since the relative positions of other particles may change in every simulation step, this deactivation probability evolves in the simulation.

3.2 Update scheme

The calculation of the deactivation for all the grid cells may be carried out as a whole in a simulation step, according to the distribution of the particles. This results in a easier implementation, since all the information needed will be gathered in a single simulation step, and after being used for the step calculation of the particles it will be casted away.

A somehow not-so-straightforward implementation would necessitate the update of the local grid topology (in terms of grid cell deactivation) immediately

6. The particle makes the position (i, j) unavailable for other particles. This unavailability can be understood as a deactivation probability for 1.

7. Out of the boundary of the grid.

8. This information is indispensable for a correct implementation. Consider the situation in which a particle's position remains unchanged in a simulation step; in the next step, this particle's potential move should not be affected by a grid cell deactivated by itself from the last step.

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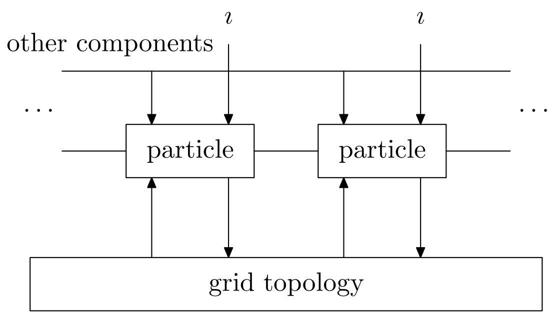

Figure 5: Sequential update scheme for the particles. The interactions with the grid cells should be carried out instantly.

after the step execution of a single particle. Particles should be processed in the simulation system sequentially. Since for each particle, only the local grid topology will be affected, this implementation variant has the same computational complexity. The difference lies in that the for the actual flowing particle, the information or state change of its neighbour particles is always up-to-date. (State change of a particle can take place within the same simulation step, that is, after being processed in a simulation step, the new state of a particle may exert influence on the particles to be processed in the same simulation step.) This fact should not be neglected when the particles are considered as a flow, we illustrate this in Figure 5.

3.3 Flow direction revisited

$\longrightarrow$

?	?	–
?		–
?	?	–

Figure 6: Deactivating grid cells in case of a given flow direction (represented by the arrow). The central position is inaccessible for other particles; for those grid cells associated with "?" the deactivation probability should be computed. Grid cells associated with "–" may be considered as irrelevant.

We have mentioned earlier that a particle affects its immediate neighbours (2), in terms of grid cell deactivation. Another interesting phenomenon is that, when the particles flow in a stable direction, we may even restrict this set of relative positions to a smaller set (that is, a set of relative positions against the flow direction): the relative positions indexed $(1, 1)$, $(1, 0)$ and $(1, -1)$ become less important, they do not concern those particle already sequentially processed in the current simulation step or the current particle itself, for the other particles to be processed, grid cell information on these locations may be renewed later in the current simulation step. Cf. Figure 6.

This feature has been confirmed by our simulation experiments.

4 A Test Case Investigation

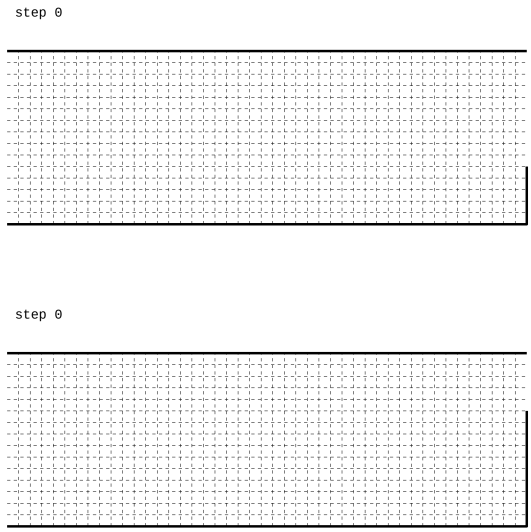

Figure 7: Different α-ratios. To modify the flow-out capacity, the sink (on the right side) has been partially closed.

In this section, we give an implementation of a concrete test case. Consider the flow of particles with different α-ratios (cf. (1)) for $\frac{3}{2}$ and 3 respectively, as shown in Figure 7.

We apply the simplest step choice rules for the particles. Whenever possible, a particle flows in the direction from its current position toward the sink. If a particle experiences no position change (due to congestion etc.) in the last simulation step, it tries a random step to its immediate neighbours (cf. (2)). We also assume that the index $\imath$ evolves continuously from the source ($\imath_{\text{in}}$) to the sink ($\imath_{\text{out}}$) in the first dimension of the grid, i. e. $\imath$ will be computed columnwise; we have argued earlier that it would not be necessary to

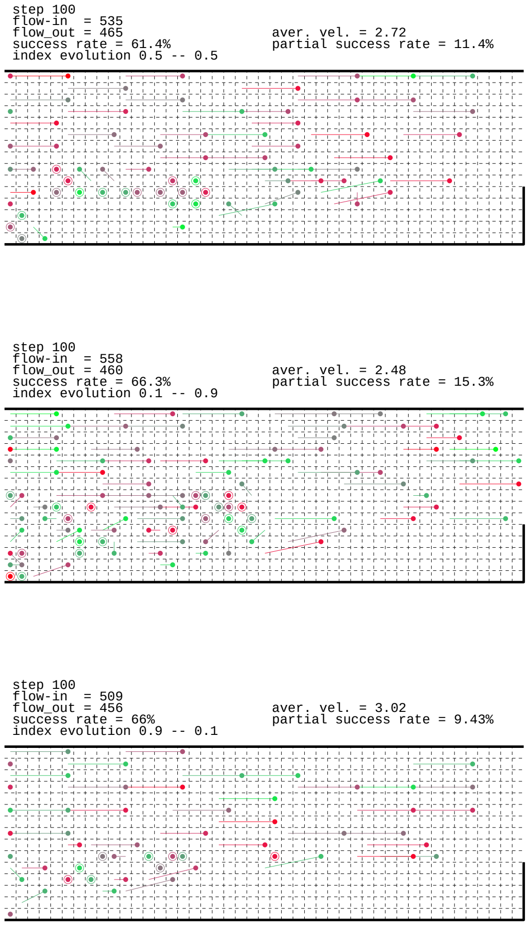

Figure 8: Simulation results after 100 steps for $\alpha = 1.5$ with different $\imath$ evolution. Particles are produced with a probability of 0.667 at the source.

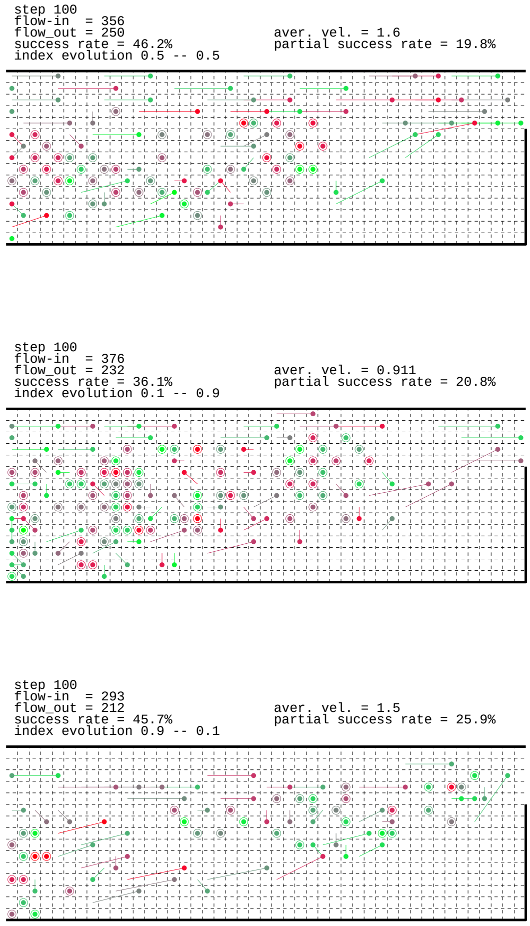

Figure 9: Simulation results after 100 steps for $\alpha = 3$ with different $\imath$ evolution. Particles are produced with a probability of 0.667 at the source.

consider the change of $\imath$ in the second dimension.

Some random simulation test results are given in Figures 8 and 9. The particles are produced from the source with a probability of 0.667. This is a somehow large production probability chosen to demonstrate the unbalanced flow. On the other hand, for the particles (produced with this probability), entrance into the system may be denied due to the deactivated grid cells in the first column of the two-dimensional grid. So this probability is always smaller than the actual flow-in density. Larger discrepancy is to be expected for larger production probability.

Particles are drawn in a random colour, particles with no position change are drawn with a second circle. In Figure 8 slight congestions have been observed, but mainly due to simple step choice, i. e. congestions take place as a local "bottle neck" phenomenon where some of the particles take steps with non-zero vertical components and thus collide with other flowing particles. In contrast, in Figure 9, serious congestions have been observed, due to the actual unbalanced flow-in and flow-out capacities. The evolution of the $\imath$ indices can be recognized by the local distribution of the particles (Recall: low $\imath$ implies that the particles claim relatively small planar space. For example, in Figure 9, when $\imath$ keeps constant as 0.5, the particles are roughly unanimously distributed, whereas when $\imath$ changes from 0.1 to 0.9, particles are distributed more loosely on the right then on the left. Figure 8 does not exhibit such differences, because the flow is almost balanced.)

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The Figures 8 and 9 present only static pictures. On the web page `http://www.math.tu-berlin.de/~chenmin/misc/index.html` we provide some full test results in PDF format and AVI format (encoded using the open source media player "MPlayer"[9]) which can be viewed as "motion pictures".

5 Further Discussion

The current paper discusses the modelling possibility of pedestrian flow with unequal flow-in and flow-out capacities. Common approaches targeting this problem concentrated on the so-called congestion formation. However, another aspect of the problem, namely, the compressibility of the flow—not in the strict sense of classical physics, but at least to certain extent—has often been neglected in grid-based methods. In this category, common CA models describe the state change of a grid cell by a set of configuration (also called transition) rules. These rules are then to be applied on the grid cells in a neighbourhood of a cell at a given position. In the model of the current paper, besides these rules by which a grid cell's state can be decided, we also consider a grid cell's state of being deactivated that the accessibility of this cell may be denied. This deactivated state of a cell can be affected by multiple particles in a neighbourhood, and reversely, the step choice in the flowing process of a single particle takes into consideration the states of multiple surrounding cells of the current particle. By checking this deactivation state of the grid cells for accessibility, the compressibility of the pedestrian flow can be simulated. This method of modifying the grid cell in a virtual way offers a first solution basis of describing various real world scenarios. Real human pedestrians may react to environmental settings very differently. This is partly shown that they make different claims on physical spaces. In a broader sense, describing these issues would be possible, once sufficient statistical data become available to construct the neighbourhood (not necessarily the fixed form of (2)) and the relevant p functions reasonably.

In a second paper [4] we considered the more general situation in which the flow direction of the particles is not necessarily a straight line. Apart from approximating the underlying simulation geometry on a regular grid, we applied a two-dimensional curvilinear grid, once the flow direction is definable as a curve. In the same work, linking and crossing of paths were considered as singular points (where a curve becomes non-differentiable).

In both papers, we have tried to restrict the model itself to the simplest form, independent of other methods. A further combination of different models is intrinsically feasible. So, technically this enables the embedding of small, local simulation into large systems. The current paper and [4] may serve as two base computational modules for complex systems of pedestrian flow simulation.

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