Technische Universität Berlin SS 2017
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4. Exercise sheet - solutions

FV/FD-Methods for the solution of pde's

1) Exercise
Use characteristic l_0 length, the velocity v_c and temperature u_c to write down the convective heat conduction equation

$$c_p \rho [\frac{\partial u}{\partial t} + \nabla \cdot (\mathbf{v} u)] = \lambda \Delta u + f$$

in a dimensionless form.

Solution:
We define

$$\mathbf{V} = \frac{1}{v_c} \mathbf{v} \ , \quad \tau = \frac{t v_c}{l_c}$$

a dimensionless temperatur U, velocity $\mathbf{V}$, length X and a time τ. Then we get

$$c_p \rho u_c \frac{l_c}{v_c} [\frac{\partial U}{\partial \tau} + \nabla^* \cdot (U \mathbf{V})] = u_c \frac{1}{l_c^2} \lambda \Delta^* U + f$$

or

$$\frac{\partial U}{\partial \tau} + \nabla^* \cdot (U \mathbf{V}) = \frac{\lambda}{c_p \rho l_c v_c} \Delta^* U + \frac{l_c}{c_p \rho v_c u_c} f \ .$$

With the temperature conduction number $a = \frac{\lambda}{c_p \rho}$, the Prandtl-number

$$Pr = \frac{\nu}{a}$$

and the dimensionless heat source $F = \frac{l_c}{c_p \rho v_c u_c} f$ we get at the end

$$\frac{\partial U}{\partial \tau} + \nabla^* \cdot (U \mathbf{V}) = \frac{1}{Pr \, Re} \Delta^* U + F \ .$$

2) Exercise
Discretize the Stokes equation

$$-\nu \Delta \mathbf{v} + \nabla p = f \ , \quad \nabla \cdot \mathbf{v}$$

on the region $\Omega =]0,1[\times]-1,0[$ with the boundary conditions for $\mathbf{v} = (u, v)$:

$$u = 1 \ , \ v = 0 \quad \text{on} \quad]0,1[\times \{0\} \quad \text{and} \quad \mathbf{v} = \mathbf{0} \ \text{ on the other boundary of } \Omega \ ,$$

with a finite volume method on staggered grids and solve the problems for 20×20 and 30×30 grids.

Solution:
t.b.a.