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3. Exercise sheet

FV/FD-Methods for the solution of pde's

Discussion: 23.6.17-27.6.17

1) Exercise
Solve the equation

$$u_t = u_{xx} + u_{yy} \ .$$

numerically on

$$\Omega =]0,\pi[\times]0,\pi[$$

with homogeneous boundary conditions, and the initial condition

$$u(x,y,0) = \begin{cases} 1 & \text{for } |x-\frac{\pi}{2}| < \frac{1}{2} \text{ and } |y-\frac{\pi}{2}| < \frac{3}{2} \\ 1 & \text{for } |x-\frac{\pi}{2}| < \frac{3}{2} \text{ and } |y-\frac{\pi}{2}| < \frac{1}{2} \\ 0 & \text{otherwise} \end{cases} \ .$$

Use an appropriate finite difference discretization of Ω and test implicit and explicit time discretizations. Follow the solution to a steady state ($\frac{\partial u}{\partial t} \approx 0$).

2) Exercise
We consider the inital boundary value problem

$$\begin{aligned} u_t = u_{xx} \quad & \text{in } \Omega =]0,1[\\ u(0,t) = u(1,t) &= 0 \\ u(x,0) &= \sin(\pi x) \end{aligned}$$

with the exakt solution

$$u(x,t) = e^{-\pi^2 t}\sin(\pi x) \ .$$

Use the horizontal method of lines to solve the problem numerically and compare the numerical solutions to the exact solution. Use an equidistant finite difference discretization of u_{xx} and Ω.
Test different ode-solver including Euler-explicit and Euler-imlicit and MATLAB ode45 or an equivalent Octave-ode-solver.

3) Exercise
The heat conduction number and the temperature conduction number of a potato are

$$\lambda = 0,16\frac{W}{m\,K} \ , \quad a = 5,6\cdot 10^4\frac{m^2}{3600\,s}$$

and the convective heat transfer coefficient is in some sense uncertain, we suppose it as

$$\alpha = 30\frac{W}{m^2\,K} \ .$$

The potato is supposed as a ball Ω with the radius $R = 5\,cm$. We consider such a potato, which has after cooking the homogeneous temperature of $373\,K$. The potato was cooled in a big can with water of a constant temperature of $u_\infty = 291\,K$. The heat conduction equation is

$$u_t = a(u_{xx} + u_{yy} + u_{zz}) \ ,$$

the initial condition is

$$u(x, y, z, 0) = 373\,K \ .$$

On the boundary $\Gamma = \partial\Omega$ we consider a bc of third kind

$$-\lambda \frac{\partial u}{\partial \vec{n}} = \alpha(u - u_\infty) \ ,$$

where $\frac{\partial}{\partial \vec{n}}$ is the derivative in the outer normal direction (r-direction). Determine the minimal time when the maximal temperatur of the potato is less or equal to $333\,K$.
Because of the radial symmetry it is useful to understand this problem as a 2,5d problem (2d in polar coordinates).