Technische Universität Berlin SS 2017
Institut für Mathematik
Prof. Dr. G. Bärwolff
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2. Exercise sheet

FV/FD-Methods for the solution of pde's

Discussion: 12.6.15-16.6.16

1) Exercise
Show the benefit of the Kronecker product of matrices to implement a coefficient/stiffness matrix for the Poisson problems of the exercises of exercise sheet Nr. 1.

2) Exercise
Solve the equation

$$-u_{xx} - u_{yy} = (2 - \sqrt{x^2 + y^2})(1 - \sqrt{x^2 + y^2})y/\sqrt{x^2 + y^2}\ .$$

on

$$\Omega = \{(x, y)\,|\, 1 < x^2 + y^2 < 4\}$$

with homogeneous boundary conditions.

3) Exercise
Solve the steady state heat conduction boundary value problem

$$\begin{aligned} -\Delta\, u &= 0\ , && \text{in } \Omega = \{(x, y, z)|x^2 + y^2 < 1,\ 0 < z < 2\} \\ u &= 1\ , && \text{on } \Gamma_1 = \{(x, y, 0)|x^2 + y^2 < 1\} \\ \frac{\partial u}{\partial n} &= 0\ , && \text{on } \Gamma_2 = \{(x, y, z)|x^2 + y^2 = 1, 0 < z < 2\} \\ \frac{\partial u}{\partial n} &= 1\ , && \text{on } \Gamma_3 = \{(x, y, 2)|x^2 + y^2 < 1\}\ . \end{aligned}$$

$\frac{\partial}{\partial n}$ means the outer normal directional derivative on Γ.

4) Exercise
Construct the mapping $\Psi : \{1, 2, \ldots, N\} \times \{1, 2, \ldots, M\} \to \{1, 2, \ldots, N * M\}$ which maps the position (i, j) of a rectangular matrix A of type $N \times M$ to the relevant index ind of a vector $\vec{a} \in \mathbb{R}^{N\times M}$ which contains row by row of A, and its inverse Ψ^{-1}.

5) Exercise
The 2d coefficient matrix A of the Poisson problem $-\Delta\, u = f$ on a quadratic domain Ω with Dirichlet boundary conditions we can create using the Kronecker-product

$$A_2 = I \otimes A + A \otimes I$$

with $A \in \mathbb{R}^{(N-1)\times(N-1)}$

$$A = \begin{pmatrix} 2 & -1 & 0 & 0 & \ldots & 0 \\ -1 & 2 & -1 & 0 & \ldots & 0 \\ & & & \ddots & & \\ 0 & \ldots & 0 & -1 & 2 & -1 \\ 0 & \ldots & 0 & 0 & -1 & 2 \end{pmatrix}$$

and the identity $I \in \mathbb{R}^{(N-1)\times(N-1)}$.
Show the formula

$$(C \otimes B)(v \otimes w) = Cv \otimes Bw$$

and use it to determine the eigenvalues of A_2 when A has the eigenvalues $\lambda_1, \ldots, \lambda_{N-1}$.