Technische Universität Berlin SS 2017
Institut für Mathematik
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Projects

FV/FD-Methods for the solution of pde's

Project 1)

Construct a Finite-Volume scheme to solve the 3D heat-conduction initial-boundary value problem 3) of exercise sheet 3 in spherical coordinates for Dirichlet bc and for the bc of problem 3) of sheet 3. Determine the numerical steady state solution for the Dirichlet bc

$$u(x,y,z) = 263\,K + \sin(\arccos \frac{z}{\sqrt{x^2+y^2+z^2}})45\,K$$

on the boundary $\Gamma = \{(x,y,z)\,|\,x^2+y^2+z^2 = 25\,cm^2\}$.
Validate your approximation by solving the 1D problem of sheet 4 with the fully 3D scheme.

Project 2)

The Ω should be the union of

$$\Omega_1 =]0,2[\times]0,1[\;, \quad \Omega_2 = [2,6[\times]0,1[\quad \text{and} \quad \Omega_3 =]2,6[\times]1,3[$$

minus the points (2,1) and (2,0). This is a so called L-shaped region. On the boundary $\Gamma_1 = \{0\}\times]0,1[$ a parabolic velocty profile is given. At the boundary $\Gamma_2 = \{6\}\times]0,3[$ for the velocity $\vec{v} = (u,v)$ we have $\frac{\partial u}{\partial x} = 0$ and $v = 0$ as boundary conditions. On the other boundary $\Gamma_3 = \partial\Omega \setminus (\Gamma_1 \cup \Gamma_2)$ we have $\vec{v} = \mathbf{0}$.

The velocity field should be determined by the solution of the velocity potential equation

$$\Delta\Psi = 0$$

where the stream function Ψ and the velocity components u and v are connected by

$$\frac{\partial \Psi}{\partial y} = u \qquad -\frac{\partial \Psi}{\partial x} = v\;.$$

This results in a potential flow.

On Ω we consider the concentration transport equation

$$\frac{\partial c}{\partial t} + \nabla \cdot (c\vec{v}) = D\Delta c$$

where c ist the concentration of a pollutant and D ist a diffusion coefficient. On Γ_1 for the pollutant we have the Dirichlet boundary condition $c = 1$. On the rest of the boundary we have homogenuous Neumann boundary condition for c, i.e. $\frac{\partial c}{\partial \nu} = 0$

with the outer boundary normal ν. At time $t = 0$ the pollutant has the initial value $c = 0$ in Ω.

Construct a FD or a FV method to determine the velocty field $\vec{v}$ and after that dicretize the initial boundary value problem for the pollution transport and test it for different values of the diffusion coefficient.

Project 3)

Analyse the solution behaviour of the linear equation systems which follows by the solution of Poisson-like problems and convection-diffusion problems. Compare iterative methods to cg-methods and investigate the matrix properties like condition number, eigenvalues, convergence rate and so on.

Project 4

Discretize the Stokes equation

$$-\nu\Delta\mathbf{v} + \nabla p = f \ , \quad \nabla \cdot \mathbf{v}$$

on the region $\Omega =]0,1[\times]-1,0[$ with the boundary conditions for $\mathbf{v} = (u, v)$:

$$u = 1 \ , \ v = 0 \quad \text{on} \quad]0,1[\times\{0\} \quad \text{and} \quad \mathbf{v} = \mathbf{0} \ \text{ on the other boundary of } \Omega \ ,$$

with a finite volume method on staggered grids and solve the problems for 20×20 and 30×30 grids.

Try to extend the method to the Navier-Stokes equation

$$(\mathbf{v} \cdot \nabla)\mathbf{v} - \nu\Delta\mathbf{v} + \nabla p = f \ , \quad \nabla \cdot \mathbf{v}$$

for small Reynolds-numbers.