The one-sided exit problem for integrated Lévy processes

Frank Aurzada

TU Berlin

joint work with Steffen Dereich (Marburg)

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Outline

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Statement of the problem

Given: $(A_t)_{t\geq 0}$ stochastic process with $A_0 = 0$.
Goal: Find asymptotics of

$$\mathbb{P}\left[\sup_{0\leq t\leq T} A_t \leq 1\right] \approx \ ?\,, \qquad \text{as } T \to \infty.$$

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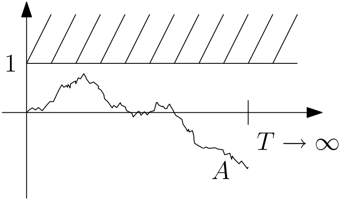

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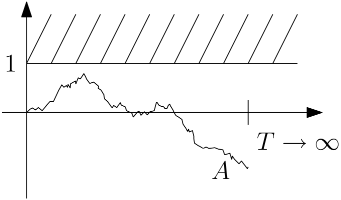


Typically, one expects

$$\mathbb{P}\left[\sup_{0\leq t\leq T} A_t \leq 1\right] = T^{-\theta+o(1)}, \qquad \text{as } T \to \infty$$

with $\theta > 0$, called survival exponent

Relations to other questions

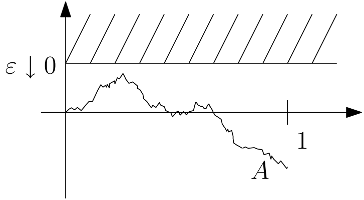
$$\mathbb{P}\left[\sup_{0\le t\le T} A_t \le 1\right] = T^{-\theta+o(1)}, \qquad \text{as } T\to\infty.$$

- statistical mechanics: Burgers' equation – a PDE considered with random initial condition (Sinaĭ'92, Bertoin'98, Molchan'99, Simon'08)
- Entropic repulsion/wetting models – discrete case (Caravenna/Deuschel'08)
- pursuit problems – 'random prisoner is followed by a random policeman' (Li/Shao'02)
- zeros of random polynomials (Dembo/Poonen/Shao/Zeitouni'02, Li/Shao'04)

Relations to other questions

for an H-self-similar processes, the question is the same as

$$\mathbb{P}\left[\sup_{0\le t\le 1} A_t \le \varepsilon\right] = \varepsilon^{\theta/H+o(1)}, \qquad \text{as } \varepsilon \to 0.$$



that is, the lower tail of $A_1^* := \sup_{t\in[0,1]} A_t$

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Known results: continuous-time processes

$$\mathbb{P}\left[\sup_{0\le t\le T} A_t \le 1\right] = T^{-\theta+o(1)}, \qquad \text{as } T\to\infty.$$

- Brownian motion: $\theta = 1/2$ (reflection principle gives even the law)
- $A_t = \int_0^t B_s \,\mathrm{d}s$ integrated Brownian motion: $\theta = 1/4$ (McKean'63, Goldman'71, Sinaĭ'92)
- fractional Brownian motion: $\theta = 1 - H$ (Molchan'99)
- Lévy processes (LP) (classical results of fluctuation theory)
- many Gaussian processes: polynomial scale (Li/Shao'04)
- integrated stable LP with no negative jumps (Simon'07)

Known results: discrete case

$$\mathbb{P}\left[\sup_{1\le n\le T} A_n \le 1\right] = T^{-\theta+o(1)}, \qquad \text{as } T \to \infty.$$

- $A = X$ random walk: e.g. $\theta = 1/2$ for $\mathbb{E}X_1 = 0$ (classical)

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  - integrated *simple* RW: $\theta = 1/4$ (Sinaĭ'92)
  - IRW with finite exp. moments: $\theta \le 1/2$ and logarithmic upper bound (Caravenna/Deuschel'08)
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  - IRW (lattice valued, other special cases): $\theta = 1/4$ (Vysotsky'10)

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    Our work: true for general IRW with finite exp. moments!

Outline

Main results: overview

three main results:

- (1) universality of the asymptotics
- (2) existence of the survival exponent
- (3) robustness concerning the barrier

(1) Universality result

- X – either a LP or RW with $\exists \beta > 0$: $\mathbb{E} e^{\beta |X_1|} < \infty$ and $\mathbb{E} X_1 = 0$

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Theorem

For two processes X and Y as above we have

$$\mathbb{P}\left[\sup_{0\leq t\leq T} \mathcal{I}(X)_t \leq 1\right] \asymp_{\log} \mathbb{P}\left[\sup_{0\leq t\leq T} \mathcal{I}(Y)_t \leq 1\right]$$

$f \asymp_{\log} g$ means $(c \log T)^{-\delta} f(T) \leq g(T) \leq (c \log T)^{\delta} f(T)$, for some $c, \delta > 0$ and large enough T.

(1) Universality result: Main example

Fractional integration operator:

$$\mathcal{I}_\alpha(X)_t := \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} X_s \,\mathrm{d}s, \qquad t \geq 0$$

for some $\alpha > 0$ (recall: $\mathcal{I}_\alpha = (\mathcal{I}_1)^\alpha$ for integer α).

Corollary

For two processes X and Y as above we have

$$\mathbb{P}\left[\sup_{0\leq t\leq T} \mathcal{I}_\alpha(X)_t \leq 1\right] \asymp_{\log} \mathbb{P}\left[\sup_{0\leq t\leq T} \mathcal{I}_\alpha(Y)_t \leq 1\right]$$

In particular, the asymptotics are equivalent w.r.t. $\asymp_{\log}$!

(1) Universality result: Integrated random walk

Usual integration operator:

$$\mathcal{I}_1(X)_t = \int_0^t X_s \,\mathrm{d}s, \qquad t \geq 0$$

Corollary

For any either X a LP or RW with $\exists \beta > 0$: $\mathbb{E}e^{\beta|X_1|} < \infty$ *and* $\mathbb{E}X_1 = 0$.

$$\mathbb{P}\left[\sup_{0\leq n\leq T} \sum_{i=1}^{n} X_i \leq 1\right] \asymp_{\log} \mathbb{P}\left[\sup_{0\leq t\leq T} \int_0^t X_s \,\mathrm{d}s \leq 1\right]$$

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(1) Universality result: More precise formulation

- X – either a LP or RW with $\exists\beta > 0$: $\mathbb{E}e^{\beta|X_1|} < \infty$ and $\mathbb{E}X_1 = 0$.
- Integration operator:

$$\mathcal{I}(X)_t = \int_0^t K(t-s)X_s \,\mathrm{d}s, \qquad t \geq 0$$

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Theorem

For a process X as above and a Brownian motion B

$$(c \log T)^{-2(1+\alpha)} \leq \frac{\mathbb{P}\left[\sup_{0 \leq t \leq T} \mathcal{I}(X)_t \leq 1\right]}{\mathbb{P}\left[\sup_{0 \leq t \leq T} \mathcal{I}(B)_t \leq 1\right]} \leq (c \log T)^{2(1+\alpha)}$$

(2) Existence of the survival exponent

- X – either a LP or RW with $\exists\beta > 0$: $\mathbb{E}e^{\beta|X_1|} < \infty$ and $\mathbb{E}X_1 = 0$.
- fractional integration operator ($\mathcal{I}_0 := \mathrm{Id}$):

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Theorem

There is a non-increasing function $\theta : [0, \infty) \to (0, 1/2]$ such that for any process X as above

$$\mathbb{P}\left[\sup_{0 \leq t \leq T} \mathcal{I}_\alpha(X)_t \leq 1 \right] = T^{-\theta(\alpha) + o(1)}.$$

In particular, $\theta(0) = 1/2$ and $\theta(1) = 1/4$.

(2) Existence of the survival exponent: boundedness

$$\mathcal{I}_\alpha(B)_t = \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} B_s \, \mathrm{d}s, \qquad t \geq 0$$

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The function $\theta : [0,\infty) \to (0, 1/2]$ *is non-increasing,* $\theta(0) = 1/2$, $\theta(1) = 1/4$ *and*

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The constant b actually has a relation to the question of random polynomials having no real zeros (studied by Dembo et al.'02):

$$\mathbb{P}\left[\sum_{i=0}^{2n} \xi_i x^i < 0 \;\; \forall x \in \mathbb{R}\right] = n^{-4b+o(1)}, \qquad n \to \infty$$

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Except $\theta(0) = 1/2$ and $\theta(1) = 1/4$, no other values are known. Even $\theta(2)$ is unknown:

$$\mathbb{P}\left[\sup_{0\leq t\leq T} \int_0^t \int_0^s B_u \,\mathrm{d}u \,\mathrm{d}s \leq 1\right] = T^{-\theta(2)+o(1)}.$$

(2) Survival exponent: comparison to FBM

Recall that $\mathcal{I}_\alpha(B)$ and FBM B^H with $H = \alpha + 1/2$, $\alpha \in [0, 1/2]$ are closely related: with an independent, very smooth process M^H,

$$B^H = \mathcal{I}_\alpha(B) + M^H$$

Theorem (Molchan'99, A.'10+)

For fractional Brownian motion we have, for some $c > 0$,

$$(\log T)^{-c}\, T^{-(1-H)} \le \mathbb{P}[\sup_{0\le t\le T} B_t^H \le 1] \le (\log T)^{c}\, T^{-(1-H)},$$

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The survival exponents of α-times integrated BM $\mathcal{I}_\alpha(B)$ and FBM B^H with $H = \alpha + 1/2$ do not coincide, at least for $\alpha > 1/4$, i.e. $H \in (3/4, 1]$.

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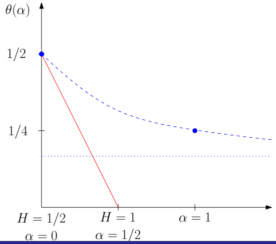
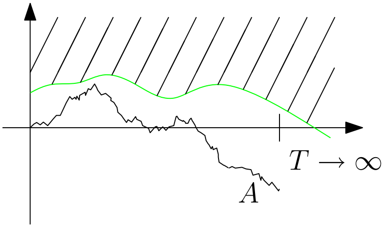


(3) Robustness concerning the barrier: motivation

Asymptotics with respect to different barriers:

$$\mathbb{P}\left[\forall t \in [0,T] \,:\, A_t \le B(t)\right]$$

When can one replace B by $B \equiv 1$?



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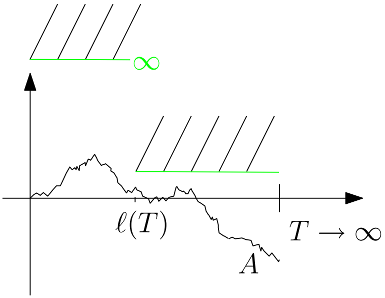

Goal: understand which scenario gives the main contribution to the probability.

(3) Robustness concerning the barrier: exemplary result

Theorem

Let A be a Gaussian process and g be from the RKHS (Cameron-Martin space) of A. Then

$$\mathbb{P}\left[\sup_{0\le t\le T} A_t \le 1\right] = T^{-\theta+o(1)} \quad \textit{iff} \quad \mathbb{P}\left[\sup_{0\le t\le T} (A_t + g_t) \le 1\right] = T^{-\theta+o(1)}.$$

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Also, upper (lower) bounds imply upper (lower) bounds.

Structure of the RKHS of $I_\alpha(B)$:
g in RKHS iff ex. $f' : [0,\infty) \to \mathbb{R}$ s.t. $\int_0^\infty f'(s)^2 \, \mathrm{d}s < \infty$ and

$$g(t) := \frac{1}{\Gamma(\alpha+1)} \int_0^t (t-s)^\alpha f'(s) \, \mathrm{d}s, \qquad t \ge 0.$$

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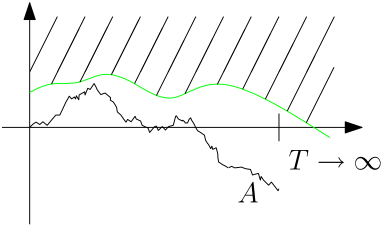

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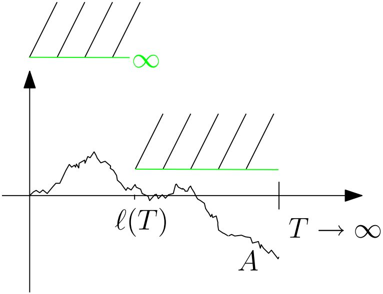

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Coupling of the LP/RW with BM via KMT

Main idea for the universality result

Coupling of the LP/RW with BM via KMT

- Komlós/Major/Tusnády'75: one can couple a LP/RW X with a BM B such that

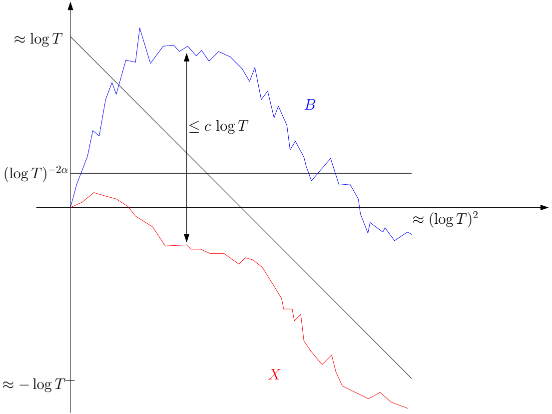
$$|X_s - B_s| \leq c \log T \qquad \text{for all } 0 \leq s \leq T$$

  with very high probability.
- problem: $|X_s - B_s| \approx \log T$ may happen at the beginning (for small s), which adds up too much error when integrating:

$$\int_0^t X_s ds \leq \int_0^t B_s ds + ct \log T \leq 1 + t \log T$$

$$\mathbb{P}\left[\int_0^t B_s ds \leq 1, \forall t \leq T\right] \leq \mathbb{P}\left[\int_0^t X_s ds \leq 1 + t \log T, \forall t \leq T\right]$$

- Make the process X behave as follows:



- behaviour of X costs only a logarithmic probability... (one has to use a decoupling argument, FKG-type inequality)
- After $\approx (\log T)^2$, the process can be estimated by B using the coupling.

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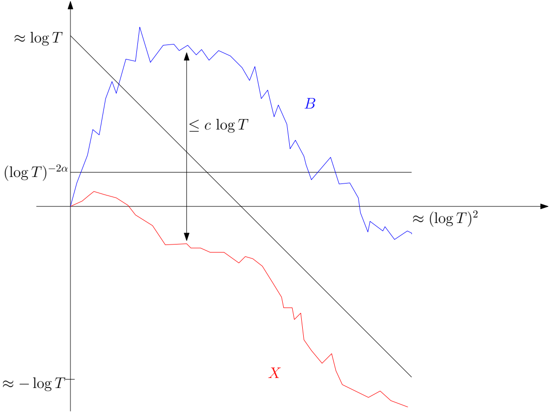

$$\mathbb{P}\left[\int_0^t X_s ds \le 1, \forall t \le T\right] \ge \mathbb{P}\,[\text{construction}]\,\mathbb{P}\left[\int_0^t B_s ds \le 1, \forall t \le T\right]$$

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Open problems

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$$\mathbb{P}\left[\sup_{0 \le n \le T} S_n \le 0 \,\middle|\, X_T = 0, S_T = 0 \right], \qquad \text{as } T \to \infty \text{ ?}$$

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- int. LP/RW with finite variance (but infinite exp. moments)
- int. LP/RW with infinite variance (some considered by Simon'07), open: integrated symmetric stable LP

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- IRW bridge: e.g. $S_n = \sum_{i=i}^{n} X_i$ ISRW conditioned to return to origin

$$\mathbb{P}\left[\sup_{0\leq n\leq T} S_n \leq 0 \middle| X_T = 0, S_T = 0\right], \qquad \text{as } T \to \infty\ ?$$

- int. LP/RW with finite variance (but infinite exp. moments)
- int. LP/RW with infinite variance (some considered by Simon'07), open: integrated symmetric stable LP

Thank you for your attention!

References:

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Frank Aurzada
Technische Universität Berlin
www.math.tu-berlin.de/∼aurzada/