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Geometry I
WS 09/10

Exercise Sheet 8

Exercise 1: Cross-ratio. (4 pts)

Let l_1, l_2, l_3 be three skew lines in $\mathbb{R}P^3$. Let a, b, c, d be lines in $\mathbb{R}P^3$ which intersect each line l_i. Then the four intersection points $\{a_i, b_i, c_i, d_i\}$ on l_i determine a cross-ratio $q_i = \mathrm{cr}(a_i, b_i, c_i, d_i)$. Show that $q_1 = q_2 = q_3$.

Exercise 2: Pencil of conics. (6 pts)

Consider the following pencil of conics in $\mathbb{R}^2$:

$$\gamma_t : \; x^2 + (1-t)y^2 + 2tx - 2(1-t)y + 2 - t = 0.$$

Find $t \in \mathbb{R}$ such that:

1. γ_t is a parabola;
2. γ_t is a hyperbola;
3. γ_t is an ellipse (with real points);
4. γ_t is empty (an ellipse with no real points);
5. γ_t is a circle;
6. γ_t is a degenerate conic.

Exercise 3: Canonical form of conics. (4 pts)

1. Classify and find the canonical form of the following conics in $\mathbb{R}^2$:
   (a) $\gamma_1 : \; x^2 + 2xy + y^2 + 4x = 0$;
   (b) $\gamma_2 : \; x^2 + 6xy + y^2 - 3 = 0$;
   (c) $\gamma_3 : \; 3x^2 + 2xy + 3y^2 - 8 = 0$.
2. Write down the change of coordinates which transforms the conics γ_i in canonical form.

Exercise 4: Parabola. (4 pts)

In $\mathbb{R}^2$ consider the conic of equation $\gamma : \; 4x^2 + 4xy + y^2 + x = 0$.

1. Show that γ is a parabola and find its vertex;
2. Find the tangent lines to γ which are parallel to the line $x = 0$.

Due: Tutorial on 15.01.10