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Geometry I
WS 09/10

Exercise Sheet 6

Exercise 1: Fixed points of projective transformations. (4 pts)

1. Show that any projective transformation on $P^2(\mathbb{R})$ has a fixed point.
2. Let V be a vector space over the field $\mathbb{K}$. Give necessary and sufficient conditions for an arbitrary projective transformation (which is not the identity) to have fixed points on $P(V)$.

Exercise 2: Cubic equations. (4 pts)

Let E be subspace of $P^2(\mathbb{C})$ defined by the cubic equations

$$\begin{cases} x_0 x_1 x_2 = 0, \\ x_0^3 + x_1^3 + x_2^3 = 0. \end{cases}$$

1. How many points are in E?
2. Let $a, b \in E$, $a \neq b$. Prove that the line through a and b contains $c \in E$, $c \neq a, b$.

Exercise 3: Duality. (6 pts)

Let $A = [1, 0, 0]$, $B = [0, 1, 0]$, $C = [0, 0, 1]$, $D = [1, 1, 1]$ be points in $P^2(\mathbb{R})$. Let $A' = AD \cap BC$, $B' = BD \cap CA$ and $C' = CD \cap AB$. Let $X = BC \cap B'C'$, $Y = CA \cap C'A'$ and $Z = AB \cap A'B'$

1. Describe the dual of A, B, C, D.
2. Show that X, Y, Z are collinear.
3. Find the dual statement corresponding to that you have proved in 2.

Exercise 4: Dual of projective points. (4 pts)

Let V be a 3-dimensional vector space with basis v_1, v_2, v_3 and let A, B, C be points in $P(V)$ expressed in homogeneous coordinates relative to this basis by

$$A = [2, 1, 0], \quad B = [0, 1, 1], \quad C = [-1, 1, 2].$$

Find the coordinates with respect to the dual basis of the three points in the dual space $P(V')$ which represent the lines AB, BC and CA.

Due: Tutorial on 4.12.09